

*Report of the Exchange Programme of
ILIP., St. Petersburg and
Eubers Venner, Aarhus.*

*Part 2: The Stay in Denmark seen with Russian Eyes.
September 13-20 1992.*



*ILIP. and
University of Electrical Engineering
St. Petersburg.*

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Preface

This is part 2 of the report of the first half of the exchange program between ILIP, St. Petersburg and Eulers Venner, Aarhus, written by the Russian participants.

Because of the general situation in Russia it was not possible for our Russian friends to have this report printed in St. Petersburg. We therefor took upon us the task of printing it for them. The contents is not changed, except for changes caused by differences in the word processing systems; and the front page is produced by the Danes.

On behalf of the Danish participants

Pieter van de Griend

Lars P. G. Westergaard

Eva Kathrine Petersen

Introduction

Here we are happy to give our notes of what happened to us in the University of Aarhus and the area round during our staying in the wonderful city of Aarhus. We, a group of Russian students in mathematics took part in the exchange program between Eulers Venner and ILIP (Innovations of Leningrad Institutes and Enterprises), which we were connecting Danish Students through and which will help to arrange a program for them.

Last October Thomas Christensen, a student of the Aarhus University and active participant of Eulers Venner was visiting St. Petersburg and got to know ILIP. After some time since he left St. Petersburg ILIP received a pack of proposals on behalf of Eulers Venner concerning possible exchange. We would very much thank Thomas Christensen and Jakob Grove on behalf of Eulers Venner for their initiative and all Danish students for their thoughtful program prepared for us.

A key point which made our coming possible was good will of Danish side to pay for our transportation. Additionally each of us was given 350 DKr of pocket money. From our preliminary correspondence we got to know that Democracy Fund helped it financially. We take a chance to thank them very much for their precious help which allowed us to experience many fruits of Danish democracy. No doubt this trip was very useful for us both professionally and educative and will have response on our life in Russia.

We also give our thanks to all people and institutions who were receiving us:

lecturers of the Institute of Mathematics

Jørgen Tornehave

Jørgen Hoffman Jørgensen

Johan Dupont

Johan P. Hansen

dean Karl Pedersen

students to talk to us on the University affairs

City Hall

Cathedral

Art Museum

Moesgaard Museum

Openair museum The Old City and

Canteen of the Mathematical Department

Our special thanks to the JTAS company for that day they devoted to us and for dinner at the restaurant.

We thank ILIP for help in solving technical problems and making this trip possible for us.

We are looking forward to see our Danish friends in Russia.

Sunday 13 September 1992

Before we arrived to Aarhus this day of Sunday we had had a rather long bus trip from the Russian - Polish border. We left St. Petersburg Friday night, in the afternoon of the following day we were already sitting in the bus of the Lithuanian company. Our drivers tried to make the journey as pleasant as possible. At our requests they had many stops and we could see Warsaw for one hour, Hamburg for a couple of ours + Hamburg again and Berlin (!) on the way home. So Europe became closer to us.

Last stop we had at the frontier between Denmark and Germany. All formalities were quickly settled, we are in Denmark!

Arrival.

We were very glad to come in time, it does credit our drivers once again. In fact we came even earlier agreed time (16:00) and some Danish students didn't come yet to pick their new room-mates. As it was Sunday the University was closed and we had to knock at the doors. In a very short time we saw Niels and Thomas (some of us had already known Thomas) hurrying to open the door. It meant the beginning of the program. At the meeting room of the Mathematical Department we had a wonderful foretaste of the following week accompanied with a cup of coffee and a cake. Our hosts prepared a special edition of their newsletter Q.E.D. explaining all ins and outs of the program, also travelling cards and envelopes with pocket money. When all Danes gathered we were introduced to our personal hosts. The end of this day we were enjoying rest at the students flats. All of us were sleeping dead.

Welcome our program!

Monday 14 September 1992

Welcome by Hans Anton Salomonsen and sightseeing round the institute.

The program of the trip included visits to several interesting places in Aarhus. On Monday 14 September we visited some of them. Early in the morning we met at the university and all of us followed to the lecture-room G1 in order to hear welcome by head of mathematical institute of Aarhus, Hans Anton Salomonsen. He gave some information about Aarhus university, 'Eulers Venner' and exchange programs of 'Eulers Venner', emphasizing our program in Aarhus. He also told us some facts about history of Aarhus university. After that we divided in two groups to look around the institute and parts of university. We were demonstrated several lecture-rooms, including largest one in the university and explained how process of teaching is usually elapsing. Then some of us were showed physics department and others - department of computer science. In last department we have seen several computer labs and known of daily student's work for computers. We were told about types of operating systems using in process of educating in Aarhus university. It is 'Unix' and 'Windows' in particular. After this sightseeing we left the institute and went to the City Hall.

City Hall : Lecture on the structure of Danish Democracy.

First of all we had a lecture where was given a talking on the structure of the Danish democracy on Aarhus instance, as an ordinary municipality in Denmark. At the end of the lecture we have obtained a booklet containing substance of the lecture without going into details. This information we have known we included in the following summary:

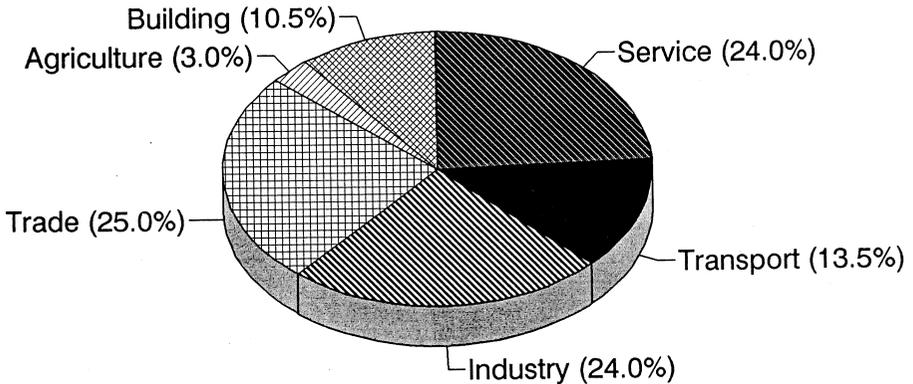
Aarhus is Denmark's second largest city with more than 260.000 inhabitants, which means that is a big city - by Scandinavian standards.

Aarhus is known as an industrial and commercial city, a city of education and culture. Aarhus is also a city with good possibilities of an active leisure time and fine experiences in it's surrounding nature.

The business community in Aarhus is not dominated by just one trade or a few companies. This means that Aarhus is independent in terms of the development and trade conditions of the different trades.

There are the three most important sectors in Aarhus economics. These are industrial production, trade and service, each accounting for about 25% of the labour force of the private sector. The distribution of employees in private sector you can see on figure 1.

fig 1. Distribution of Employees in the Private Sector.



Moreover, most of companies in Aarhus is very small. 80% of employees in the private sector work in establishments with less than 500 employees. There are only a few companies in Aarhus with more than 1000 employees. The Aarhus companies are known for producing high technology and for using the newest technology in their production. The reason being that generally the Aarhus companies are big enough to invest in new technology, but at the same time so small that they can reconvert the production and the production processes.

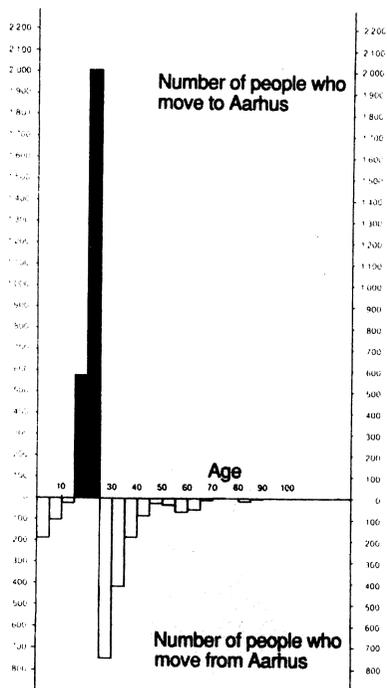
At the present Aarhus has established a municipal, Industrial Development Department, as the first municipality in Denmark.

Aarhus has good communication to the outside the world. This applies to train, car, plane or ferry; to goods as well as passenger traffic.

The best known cultural event in Aarhus is the annual Aarhus Festival, which one of the biggest cultural events of Northern Europe. The Festival takes place every year during the first week of September. The Festival offers more than 300 different arrangements covering everything from opera, theatre, classical music to video art, rock music and street performances.

Aarhus is more than a thousand years old. The vikings founded the town at the mouth of the river at the bay about the year 1000. However, the town did not really start developing until 1900, when the industrial expansion meant that people started moving from the country into town. Since 1935 the population has doubled and it is still increasing, though, in recent years the increase has showed down. With comparatively many young inhabitants Aarhus is a 'young' city. This is due to the fact that Aarhus has many schools and institutes of higher education, and that many young people come here to study. Many have to leave again when they have finished their education. It would be very interesting for you to see figure 2.

fig 2.



Moreover, there are about 120.000 residences in Aarhus. This means a little more than two persons per residence on an average. The housing standard is high. Almost all residences have a kitchen and a bathroom with a water closet. About 60% of all housing in Aarhus is for renting. The rest privately owned. 65.000 of the residences are to be found in large apartment buildings and about 35.000 are one- family houses.

Going twenty minutes to the north, south or west from Aarhus there are reach green areas. To the north and south there are woods, and to the west there is a big area with a lake and river. Furthermore, the city stands on the bay, which offers rich possibilities of activities, either on the beaches or on the bay itself. Therefore, there are many possibilities of leisure in Aarhus. There are also associations and clubs covering almost all kinds of hobbies. Besides, there is a wide choice of evening classes where all kinds of subjects are taught. The city has many resident's community centers offering many different activities such as sports, hand-work or lectures.

There are sports fields and centers everywhere in the city. The city has a golf course,

tennis courts, marinas, skating center, race course, equestrian center, rowing courses plus many other sports faculties. The Marselis Cross Country Run is without comparison the biggest annual sports event in Aarhus. More than 12.000 runners, professionals and amateurs alike, take part. Apart from this a large number of national and international sports events covering all kinds of sport take place in Aarhus every year.

Aarhus has a high social service level, also compared to the rest of Denmark. Most social service are free of charge to the citizens, but the citizens pay for these services through their taxes.

In particular, all schooling in Denmark is free. This applies to all schools from primary schools to higher education. There are more than fifty primary schools in the Municipality of Aarhus. The smallest is Ajstrup Primary School with only 56 pupils. The biggest is Skjoldhøj Primary School with more than 750 pupils. Besides, there are many private schools in Aarhus.

As for higher education, Aarhus is one of most important cities in respect of education, and it is possible to receive almost any training there. About 25.000 people study in one of the city's institutes of higher education. Furthermore, about 20.000 attend senior school, business school or vocational training.

The University of Aarhus alone has more than 13.000 students, but the city also has a school of economics, business administration and foreign languages, a technical school, a school of architecture, a school of journalism, several training colleges, a conservatory, an academy of fine arts, and many other schools and institutes of higher education. In a lot of the Aarhus institutes of higher education the teachers do research work along with teaching. Research, however, also take place elsewhere in Aarhus (at the Science Park, for instance-you can read about it below). In addition to this a large number of private companies have their research department in Aarhus.

City hall : Talking on Aarhus traditions and sightseeing round the city hall.

After first lecture we had a talking about Aarhus traditions. We were standing in front of a big blue wall. It was presenting some symbols of Aarhus. A young man with light-coloured hair was telling us about this symbols. We have learnt a lot of remarkable things. Then we followed to the wedding-room and there the young man told for customs of wedding and laws, concerning up to marriage in Denmark. We were very much surprised that marriage between two man or two women was allowed in Denmark on a level with ordinary marriages. After that we left forward upstairs. We lifted up to the roof of the City Hall. There we filled a small area. We were able to look around all of the city. Some of us have taken the pictures. We have seen such beautiful view that the memory of it will remain into our hearts forever!

Then we lifted down and followed to the room where once or twice a week Council of the City had a meeting. The Municipality of Aarhus is governed by a City Council of 31 members. The City Council is elected for a period of four years. The last municipal election took place in 1989. The City Council elects a Mayor and five aldermen.

Together the Mayor and the five aldermen form the municipal corporation. The Mayor and the five aldermen are each in charge of the six different municipal departments, which Aarhus is divided into.

Our guide gave us some information about the City Council. Here you can see the summary of it:

The Mayor Department is in charge of the municipal economy, budgets and accounts. Besides, it is in charge of personnel, industrial policy, and the supreme physical planning of the municipality. The department has about 150 employees.

The 1. Municipal Department is in charge of taxation, employment, the municipal data processing, municipal buildings and green areas of the municipality. The department has just about 150 employees.

The 2. Municipal Department is in charge of the technical sectors, roads, refuse collection and disposal, cleaning of discharge water and town planning. Furthermore, it is in charge of the fire protection service and civil defense. The department has about 1.000 employees.

The 3. Municipal Department is in charge of many tasks which demand many employees, especially minding and helping children, old, sick and handicapped people. The department has about 12.000 employees.

The 4. Municipal Department is the cultural department of the municipality. It counts about 4.200 employees, who are in charge of schools, libraries, sports, leisure activities and culture.

The 5. Municipal Department is in charge of water, heating and electricity supplies plus collective traffic. The department has just about 1.700 employees.

Music hall.

After our visit of the City Hall we set forward to the Music hall. It is situated nearly City Hall with left side. There we were met by beautiful woman-guide. She implemented our guide-tour around the Music hall. Our guide has told us of arrangements for concerts there and of the way people usually spent their time during different performances. We also have learnt that a lot of conferences was often passed at the special hall into Music hall. We visited several halls and places of interest at the Music hall. These are Big Hall for some concerts and meetings, Small Hall for conferences and little concerts, the place for the Queen of Denmark, the restaurant etc.

Moreover, we have seen the wonderful exhibition of avangard arts. Our guide has told us about the painting in the style.

Then we followed to behind the stage. There we were able to see the way arrangements for a performance elapsed. We also have seen the way the process was checked.

Cathedral.

After these visits we proceeded to the Cathedral of Aarhus. There we had a conversation about history of the Cathedral, arrangements for realization of divine service. On a level with other things we have known that the Cathedral was built about 1201, but it was destroyed in 1546 and then it was rebuilt. Every Sunday there is divine service at the Cathedral. When the talking was ended we followed to the left side of the Cathedral and then we could see allegorical and philosophical picture. There the man , (who was giving us some information at the time of our conversation), told us about the content of the picture. After that we had a sightseeing around the Cathedral and then we were walking round Aarhus-City with danish students.

Wine club of the International Student Center.

In the evening we were invited the wine club. There was tasting of some kind of wine. We were immersed to informal atmosphere, to atmosphere of friendship. Two guys told us about the ways for preparing red wine and white apples wine. We also tried such kinds of wines. From our friends we have known that apples wine was made from flowers in general with addition some apples. We spent our time there very well.

On Monday we had very meaningful and remarkable day.

Tuesday 15 September 1992

This was our second complete day of staying in Aarhus. The first day gave to us many different impressions. The most of our group went to bed rather late because we couldn't overdo the temptation of tasting the subject of the lecture we were presented in the International Student Center. We were very much attracted by the warm atmosphere of the Center and enjoyed the comfort foreign students of the Aarhus University can get here. Short night time helped us to recover in order to be ready to experience the next piece of our nice program.

Tuesday program was promising the following:

9.00-10.30: Guided tour in the Science Park

10.30-12.00: Visit to "Cryptomatics", a little company situated in the Science Park

13.15-15.00: Talks on the democratic system of the Danish Universities

15.15- : Visit to the Art Museum of Aarhus

Science Park.

At 8.45 we met in front of the Information Board and walked to the Science Park where we were welcomed by managing director Erik Jantsen. He gave us an idea of what Science Park looks like and focused our attention on the relationship between university research and industry.

In short words Science Park is an institution where science and industry meet. Mr. Jantsen helped us to understand and we could see how it happens. Science Park gives a brilliant opportunity for university and college researchers from Denmark and abroad and development personnel from the world of industry and commerce to have a place to meet together. It allows new products to be developed on the basis of up-to-minute knowledge derived from basic research combined with a practical insight into consumer demands. No surprise the concept of Science Park brings significant results in different applications such as:

- food development	- optometry
- biotechnology	- pollution & environment control
- chemistry	- medicine
- electronics	- medico-technology
- computer technology	- business administration
- telecommunications	- financial analysis

To our question on how many mathematicians work in the Science Park Erik Jantsen answered that 6 or 7 were involved. After we got a chance to see the contribution of mathematicians to the industry of data security at the example of "Cryptomatics". This company is driven by three mathematicians from the University of Aarhus and is famous

for developing the world's first home-banking system featuring a digital signature of the same documentation value as a personal signature. This has paved the way for electronic exchange of binding contracts. At the laboratory of the company we found ourselves in the world of encryption and decryption. We were told about the importance of data security in the modern world and were shown a program which provides this security. For example:

Mr.A is willing to send something like "My account number is 123". Using a key known only to him Mr.A generates a phrase which seems to be nonsense: "0+9*k#%67!3\$~& . i". But Mr.B, his partner gets the right information. All the rest who want to catch it and do not have a key for decryption will spend hundreds of years to decrypt.

We also walked through the labs of the Water Quality Institute, which is the other example of the projects developed in the Science Park. We witnessed the systems of control, monitoring and analysing of waste water which help to provide Denmark with clean water. Staying in the homes of Danish students we got an impression of Danish water to be the best in the world.

Exploring the Park we paid attention on a thoughtful way to design the building. Outside it looks a twin of the University, in spite of the fact the building was designed only in 1986. And it is very cozy and functional inside. Architects and engineers ensured that the individual rooms provide the ideal framework for developing projects. Pictures on the walls harmonize to the modernist environment of the Aarhus Park.

Afterwards, when we returned to the University we went deeper in the theoretical principles of cryptology. The lecture we were given covered the questions:

- Goals for cryptology
- History and basic concepts
- Public key systems

Danish Universities. Democratic System.

This time we had a meeting with Dean of the Mathematical Faculty Karl Pedersen who explained us how Danish universities and democratic principles come to work together. He started with a presentation of the general structure of a university at the example of the Aarhus University. Actual structure consists of three levels:

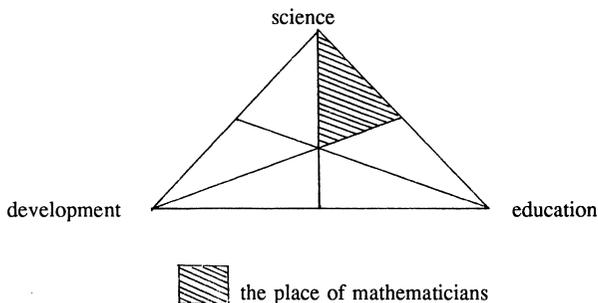
1. **University level.** Rector is a head of the University, all questions at this level are solved by Konsistorium.
2. **Faculty level.** Dean is a head of a faculty who makes decisions through a faculty board.
3. **Department = Institute level.** There is council to solve the arising questions at this level.
4. **Study Boards.**

This shows that different decisions are made on different levels. For us it was very attractive that in the decision making process participation of students is very essential. Students are able to influence either by their votes in the Study Council having 50% or by taking part in the discussions on higher levels of management like Faculty Board. This talk was followed by a fellowship of a student's activist which covered students' influence in the university democracy in theory and practice. Students elect their representatives who are involved in the decision making process. Our lecturer gave the following reasons to explain why students are the right people to this business. We entirely agree that students' candidates have:

- close contact to student environment
- better insight in current student problem
- no direct economical interests

We got the idea that to be elected student-candidate ought to have 5-10 other people to propose him. The only problem we were talked about was that there were not many students willing to sacrifice themselves for the social work.

Afterwards Johan P. Hansen (we were happy to listen to his lecture on Friday) shared with us his view on the university management. We very much liked him speaking about the relations between science, development and education. It is easy to understand the place of mathematicians in this process.



Johan P. Hansen was president of the Study Board years ago and dealt with a problem that it was very difficult to make real decisions because of the too democratic university system. That's why changes came and the actual system fits better the real life full of many unexpected situations which require quick response.

We were very interested in having listened three points of view on such key question as university management. Our kind speakers and their answers to the questions of Danish and Russian students helped us to understand "+"s and "-"s of the university democracy. We very much liked words of Johan P. Hansen that university system is rather based on the common sense than on any rules and regulations.

Visit to the Art Museum of Aarhus.

The rest of our active part of the day we spent at the Art Museum. Some of us liked pictures by British artists Gilbert & George. This is their first exhibition in Scandinavia. Their pictures very extremely large and some of them provocative. We were glad to get acquainted with the art of these two famous artist of the modern age.

Wednesday 16 September 1992

On this day we visited one of the biggest of the 5 telephone companies in Denmark. This is the Jutland Telephone Company (JTAS). There we had some lectures by mathematicians from JTAS. At the first Holger Orup gave us some information about implementation of cryptoalgorithms. Then we had a lecture on neural networks by Per Rosenbek. He presented the principles of neural networks. We have known that a neural network is a model of the human brain and that it consists of small processors are each able to make a decision. This lecture was connected with researchers on artificial intelligence and at least for us it was a remarkable lecture. After that we had the lectures by Henrik Karstoft: 'A mathematician working at JTAS' and by Jens Rasmussen: 'Network Planning'.

Lecture by Henrik Karstoft: the image recognition.

More than another we liked a lecture by Henrik Karstoft on image recognition. He told us that every system of machine vision must be able to form, to analyse and to interpret an image. It is achieved by such way:

A camera records a meaning of the light force. This information is transferred to electric signal, which has a size connected with intensity of reflecting light.

Further, this permanent electric signal is transformed to the figures. Size of the signal is checked via equal time intervals and then each meaning is transferred in to a whole number, which shows us position of considered point of chosen range of brightness. These numbers form 2-dimensional array. An element of the array is called 'the pixel'(from picture element). Then the machine must to analyse this 2-dimensional array. It is achieved by finding outline process. The algorithm finds some alterations of brightness.

Moreover, there are some problems with deepness perception, reveal of colour and so on.

Henrik Karstoft also gave some information about applications. We have learnt that this way of image recognition is applied to identify of a person fingerprints by recognition the person minutiae. He explained us the way to classify fingerprints.

Then we had an exercise in management training.

After that we followed to the restaurant and then we visited Aarhus Theatre.

After visiting the telephone company we were invited to come in the restaurant called "Jensens Bofhus" where we've eaten a lot of different things in the very nice homelike atmosphere. Then the whole gang of Russians and Danes moved to the Aarhus Theatre to enjoy the last part of the day's program - some 'Aspects of Love'.

The visit to the Aarhus theatre was anticipated by Russian students with great enthusiasm. We all knew Andrew L. Webber's music. His most popular (in St-Petersburg) opus is 'Jesus Christ Superstar' though his other works are not worse but just less known to Russian melomans. So we looked forward to that night.

When we were given the tickets we were surprised a little with the wide range of the prices and with the very democratic idea of selling more expensive tickets to the elder people.

The theatre of Aarhus is rather small and beautiful. We had seats in different places so Russian participants didn't have Danish students close enough to ask them what's going on at the scene (the musical was in Danish). Thus we were left tet-a-tet with the music and the singing. As far as we could understand the script and the title the musical was about love. That was the point where we all agreed. But if one wants to deep into the details he should simply go to Aarhus, Denmark and enjoy the musical himself.

Thursday 17 September 1992

It was the first day when we were given real lectures in mathematics by professors of the Mathematical Institute. The program for the day was as follows:

9.15-11.00: "Knots" - Lecture by Jørgen Tornehave

11.15-13.00: "Games theory" - Lecture by Jørgen Hoffman Jørgensen

13.00-14.00: Lunch

14.00- : Moesgaard Museum

Also after lunch some Russian students had meeting with scientists from the Mathematical Department working in the fields of interest to them. E.g. Serguei Gouriev met Jørgen Tind and his colleagues from the Department of Operations Research. They discussed their work, common research directions, the current state of affairs in Operations Research, exchanged addresses. Prof. Tind also gave the Russian student his articles in multicriteria optimization.

Knots by Prof. Tornehave

All the students were impressed by Prof. Tornehave's very emotional lecturing manner which still did not prevent them from enjoying the knots theory. First of all he drew a couple of complicated knots and set a problem: is it possible to determine whether one can unknot the first knot (it was so called a "clover leave" knot) or reshape it to the other one. The first question was really easy to answer (unknotable) but the equivalence of these very different-looking knots was not as obvious. So students got interested in the theory which could solve such problems.

The theory mentioned was introduced by Fenchel whose students Prof. Tornehave turned out to have been. So let K be a smooth curve in \mathbb{R}^3 . We also introduce a smooth mapping $\alpha : \mathbb{R} \rightarrow K$, with $|\alpha'(s)| = 1$ for any $s \in \mathbb{R}$. As curve is continuous $\alpha(s)$ should be periodic $\alpha(s) = \alpha(s+l)$ where l is the length of K . Then $\alpha'(s)$ is a tangent vector.

The following important concept to put forward is so called curvature of the curve. Local curvature $k(s)$ is equal to the absolute of the second derivative $|\alpha''(s)|$. Total curvature is natural to be the following integral:

$$TC = \int_0^l k(s) ds > 0$$

Then the professor shook us claiming the TC is not only nonnegative but also greater or equal to 2π . The idea of proof was to consider an unit sphere S in the tangent vector space where we have curve $K' \subseteq S$ - spherical image of K . Evidently the total curvature is just length of K' . Using spherical Crofton formula we easily get $TC \geq 2\pi$.

The next theorem was to exhibit what the TC does imply. The matter is that to be a knot the curve has to have TC of at least 4π . Moreover the TC proves to be a curve invariant. The proof was quite similar to that of the previous one.

Then the professor paid attention to the Tait conjectures - the other point of view on knots, explained relationship between knots and Jones polynomials introducing the very convenient axiomatic framework to deal with oriented links etc.

Theory of Games and Optimal Strategies by Prof. Hoffman-Jørgensen

Prof. Jørgensen showed us a technique which can be used to calculate optimal strategies in random games.

First introduce the formal concept of information. In a probability space (Ω, \mathcal{F}, P) information \mathcal{G} is a sub- σ -algebra of \mathcal{F} :

$$G \subseteq \mathcal{G} \Rightarrow G \subseteq \mathcal{F}$$

$$G_i \subseteq \mathcal{G} \Rightarrow \bigcap_{i=1}^n G_i \subseteq \mathcal{G}$$

$$G_i \subseteq \mathcal{G} \Rightarrow \bigcup_{i=1}^{\infty} G_i \subseteq \mathcal{G}$$

As time passes we get more information so we have a filter - an increasing sequence of information $\{\mathcal{F}_n\}$, $\mathcal{F}_0 \subseteq \mathcal{F}_1 \subseteq \mathcal{F}_2 \subseteq \dots$. We call a random variable X a \mathcal{G} -observable if knowledge of \mathcal{G} gives an exact value of X .

Then we consider the problem of conditional probabilities and expectations. We know that any probability depends very much on information available. In the terms introduced we can put forward the notion of a conditional expectation $E(X|\mathcal{G})$ which is the unique random variable satisfying:

$E(X|\mathcal{G})$ is \mathcal{G} -observable

$$\int_G E(X|\mathcal{G})dP = \int_G XdP \quad \forall G \in \mathcal{G}$$

Defined in this way conditional expectations are much easier to determine and to use in analysis of games. We consider random games of the following kind. Let a gambler have net fortune X_n at the moment n . Then an optimal stopping time τ is naturally to be

$$EX_\tau = \max_\sigma EX_\sigma$$

The optimal strategy can be found for finite games ($n < \infty$) and for certain kinds of infinite games, e.g. so called unimodal games with the aid of the same procedure of the backward induction. If the gambler has decided to play at most k games then it looks as follows:

$$M_k = E(X_k|\mathcal{F}_k)$$

$$M_j = E(X_j | \mathcal{F}_j) \vee E(M_{j+1} | \mathcal{F}_j)$$

Then the optimal strategy is given by

$$\tau = \inf_j M_j$$

Then the professor showed us how to calculate τ and applied the formula to the known examples e.g. quits and doubles, blackjack, hazard etc.

After lunch in the canteen of Mathematical Institute we went to the Moesgaard Museum where we saw an exhibition of Asian items collected by Danish traveller and the prehistorical man.

Friday 18 September 1992

The morning of the day of September 18th was devoted to pure mathematics with some flavor of quite unexpected application of this high science to the very practical sphere of the human activity i.e. the error correcting.

The whole day was filled with different events which one can hardly combine into something logically completed, but anyway the plan of this famous Friday was the following:

8.15–10.00: Lecture.

10.15–12.00: Lecture (of course, coffee and cakes were included).

12.15–13.00: Seminar: Division by inner factor and Embedding Theorems for Star-invariant subspaces of Hardy Spaces.

13.00–14.00: Lunch (those who attended this part of the program were delighted).

14.15–16.00: Student's movements in Denmark: Talks, arranged by two political student's organizations, regarding history, structure and purpose of the organizations.

16.15– : Evaluation of the week in Aarhus (that was quick and essence of it would be: All was great!).

17.00–17.45: Football game between team of Aarhus University and mixed company of Russian enthusiastic players and some Danish football profies. (Danes won 1:0).

Scissors Congruences and Homology

First very natural question — what is it, scissors congruences?

DEFINITION: Two polyhedra P and P' are Scissors congruences if we can decompose them into finitely many pairwise congruent pieces.

(p_i is congruent to p_i' if there exists g — isometry of \mathbb{R}^3 : $gp_i = p_i'$)

Then the following question arises: Can we decompose ANY polyhedra so they become scissors congruences?

The answer, clear and desperate, was obtained and proved by Max Dehn (1900) in form of

THEOREM: The regular cube and the regular tetrahedron of the same volume are NOT scissors congruences.

As we see, this theorem is about \mathbb{R}^3 .

If we take a plane and ask the same question the answer would be 'yes'. And it was known to Ancient Greeks as a

THEOREM: Any polygon in the plan is scissors congruent to a rectangle with the given side length.

The proof is performed as a procedure of isometric transformation of an arbitrary polygon to a rectangle with the side length l .

If we try to apply the same technique, as in plane, in \mathbb{R}^3 , we see that one extra degree of freedom ruins the construction.

But how can we prove that this is impossible using any technique?

The idea is to find some kind of invariant of the polyhedron. For example, the volume V .

It is quite clear that if $P \sim P' \Rightarrow V(P) = V(P')$ but unfortunately it's not so clear that if $V(P) = V(P') \Rightarrow P \sim P'$, and moreover, there is the 3rd Hilbert's problem which sounds like this: 'Prove that there exists 2 polyhedra with the same volume which are not scissors congruent.'

And that was the subject of the Dehn's theorem mentioned above. So we see that the volume is not the invariant we need.

Now let's introduce the tool by Dehn while proving his theorem. It's called simply and beautifully: Dehn's invariant — ψ : if P polyhedron

$$\psi(p) = \sum_{e=\text{edges}} l(e) \otimes \theta(e)$$

(for example tetrahedron)

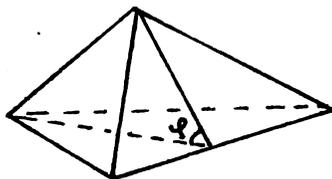
where l — length of e

θ — angle shown on the picture as

\otimes — tensor product;

$\psi \in \mathbb{R} \otimes \mathbb{T}$, where $\mathbb{T} = \mathbb{R}/\mathbb{Z} \cdot 2\pi$

$l \in \mathbb{R}$, $\theta \in \mathbb{T}$



The following theorem decides half of the problem:

THEOREM:

- 1) $P \sim P' \Rightarrow \psi(P) = \psi(P')$.
- 2) $Q1$ — regular cube, $\Rightarrow \psi(Q1) = 0$.
- 3) $Q2$ — regular tetrahedron $\Rightarrow \psi(Q2) \neq 0$

→ there is no scissors congruency between $Q1$ & $Q2$

The proof of this theorem took the biggest part of the lecture and required 4(!) additional lemmas. We wouldn't adduce it here.

— but use more resources (time, electricity etc.)

so we need compromise.

Let's take it more precise

let F_q be finite field alphabet;

$V = F_q^n$ — an n -dimensional F_q vector space.

(a_1, \dots, a_n) , $a_i \in F_q$ — message.

$C \subseteq V$ — linear subspace of V —code.

$k = \dim C$.

If we send n bit

k/n — information content (and we want it to be large!)

If aaa, bbb — code words and we get a message aba then we see that it is more close to aaa. So the main idea — code words should be very different.

Now let's define what kind of code we need. What is, in nutshell, the good code?

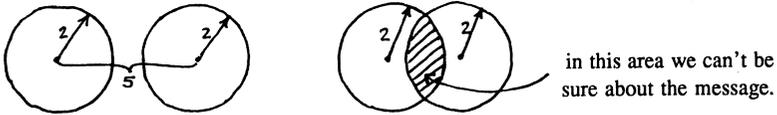
DEF weight $(a_1, \dots, a_n) =$ number of non-zero entries.

$d(V1, V2) =$ weight $(V1 - V2)$ — number of entries of $V1$ & $V2$ which are different.

Let's call at Hannig distance or simply distance.

Code is good if minimal distance of C is large. (Then it can correct many errors and that's what we want.)

For example if min dist = 5 then C can correct 2 errors:



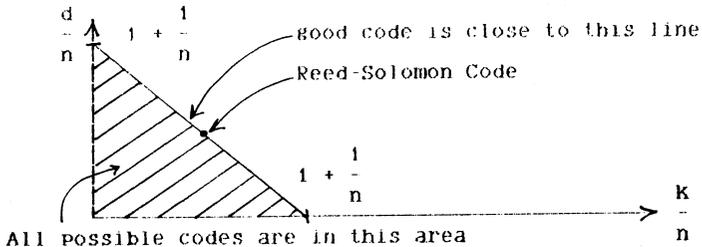
in this area we can't be sure about the message.

How large can k and d be?

The answer is given by Theorem (called Singleton bound):

$$\frac{k}{n} + \frac{d}{n} \leq 1 + \frac{1}{n}$$

So if we consider it graphically:



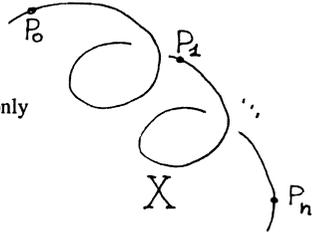
Now we know what good code is and the problem is how to construct it. That's the place where Goppas construction comes to scene.

Let X be an algebraic curve

$F_q = \{P_0, \dots, P_n\}$ – points on X .

f – a rational function on X ;

$L(dP_0)$ – a vector space consisting of all rational functions f on curve X with the only pole in point P_0 of depth of d .



And finally Goppas construction:

$L(dP) \xrightarrow{\varphi} F$, φ is a linear map

(every f is mapped into $f(P_0), \dots, f(P_n)$)

$DEFC = \varphi(L(dP)) \subseteq F_q^n$

Let's estimate parameters of C

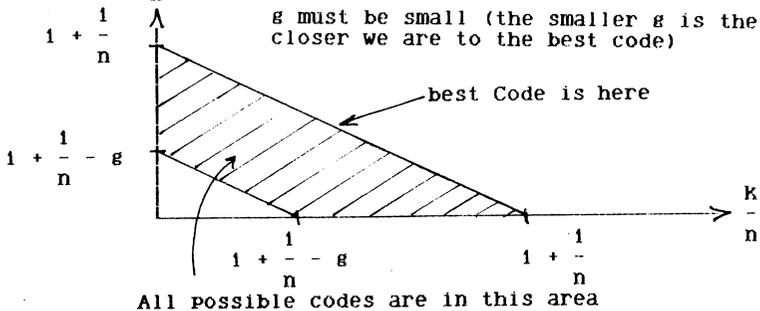
$k = \dim C = d + 1 - g$, where g is genus of X and $\min \text{dist} \geq n - d$

(All proofs are omitted.)

So we have the lower estimation of the code area:

$$\frac{k}{n} + \frac{\min \text{Dist}}{n} \geq 1 + \frac{1}{n} + \frac{g}{n}$$

or graphically:



Thus we see how Goppa's construction (very abstract) gives us the Reed-Solomon Code (very practical).

After 15 minutes' break at 12.15 K.M. Diakonov started his seminar on 'Division by inner factor and Embedding Theorems for Star-Invariant Subspaces and Hardy Spaces'.

The subject of the Seminar was very special and required deep knowledge of the topic. That was why most of the participants used this time for their own businesses.

After lunch we gathered again (for the last time) in Auditorium G1 to hear some talks about student political organizations. There we had some more spare time because

of absence of the first political organizer. This time passed in talks on non-political topics. Then the other representative came and gave us information about their structure and what they are engaged in.

After that Danish students tried to fish out of us some words of disappointment or annoyance with the whole program or some part of it. But they didn't succeed and the common opinion was: the tour was organized wonderfully and was carefully thought about so all the things together worked out very well.

The rest of the day's program was more entertaining than educational but it was not less fascinating than the first part of the day.

Resume:

The day was one of the most interesting in the tour. We were glad to learn about some interesting branches of modern science and the way of presenting them for the students.

Saturday 19 September 1992

Saturday was not as busy day as all the others and it was the one day before our departure. According to the program in the morning we had a shopping tour. So we had an opportunity to visit shops in Aarhus, acquire some goods for ourselves and some souvenirs for our friends and parents. We divided into groups. Some of us went to the small town named Bilka, some preferred visiting the city center.

The old City

In the afternoon a visit to "The Old City" - an openair museum was planned. It was designed as a small town with the streets, bridges, houses made in the ancient fashion. The museum consists of 70 old buildings which have been carefully dismantled from town all over the country and rebuilt at the museum site. "The Old City" is a museum of urban life and comprises period design from late sixteenth century to the present time. There are workshops, a chemist's shop, a school, a post-office, a custom house and an early 19th century museum. We were divided in small groups and each group had its own guide. Ours was a very nice lady, who even managed to take us inside the houses and to show us some private details of people's life during the middle ages. We visited a house of a very popular dressmaker. We learned that in that time the most important place in the lady's house was bedroom. So ladies tried to make the bedrooms very nice and comfortable, but slept in another room. We also visited the custom house, saw the place where children and their parents lived. We saw the middle-ages' washing machine. There was no running water. so they used vases for washing their hands, face, neck in the morning and it was rather complicated and rare procedure. The streets were made out of stone and might have a chance to ride the horse. So it looked like a real "back to the past".

In the evening it was time for a final party with our new Danish friends. All of us were rather tired from this busy week, but we and our friends (hope) were sorry that we are leaving. So we didn't give up and tried to make our party as solemn and merry as possible. It was not a common party, so most of us divided into groups of 4 or 8. Our hosts really tried their best. According to my own experience I can say that the food was delicious, the music was wonderful, the wine was the best I've ever tasted and the atmosphere was very friendly. But! it was time for the last good-bye and thanks because the next day was the day of our departure.

Sunday 20 September 1992

This was a very sad day of the week because we had to leave friendly Aarhus University and say good-bye to our friends. Anyway we followed the program and at 12.00 we had pizza lunch at the University.

Out-of-doors the bus was waiting us ready to repeat the way we came here. When final good-byes were said and last pictures taken we took seats in the bus with much food and plenty of impressions.

Here we once again thank all of the students and non-students for the successful program.

List of Participants

Russian Participants

1. Yekaterina Nikolayevna Mustafyna
2. Stanislav Alexandrovich Krainikov
3. Konstantin Georgyevich Egorov
4. Roman Nikolayevich Gorodnitchenko
5. Alexey Valeryevich Pertchukov
6. Yaroslav Yevgenyevich Suchkov
7. Pavel Alexandrovich Zharov
8. Vladimir Juzephovich Parkman
9. Elena Efimovna Roubintshik
10. Mikhail Victorovich Kolesnik
11. Alexey Victorovich Radionenko
12. Natalya Sergeyevna Tomashkova
13. Alexey Alexandrovich Senichev
14. Andrey Olegovich Solovyev
15. Denis Alexandrovich Prourzin
16. Olga Nikolayevna Yermilova
17. Maria Alexandrovna Victorova
18. Elena Alexandrovna Tikhonova
19. Anna Vladimirovna Podolskaya
20. Serguey Maratovich Guryev
21. Alexandr Igorevich Shramkov
22. Olga Borisovna Michailova
23. Yevgeny Borisovich Shapoval
24. Alexandra Vladimirovna Matveyeva
25. Marina Alexandrovna Robachevskaya
26. Konstantin Mikhailovich Dyakonov
27. Tatyana Vladimirovna Shulzhenko
28. Yevgenia Albertovna Safarova

Danish Participants

1. Pieter van de Griend
2. Morten Brun
3. Nils Byrial Andersen
4. Thomas Østergaard
5. Anette S. Petersen
6. Thorkild Jensen
7. Terese M. O. Nielsen
8. Christian Schlichtkrull
9. Bjarne Aagaard
10. Christian Holst Hansen
11. Henrik Just
12. Morten Storm
13. Søren Fournais
14. Jesper Kornerup
15. Lars P. G. Westergaard
16. Iver M. Ottesen
17. Anne Kirstine Simonsen
18. Elise Christiansen
19. Diana Nielsen
20. Niels Henriksen
21. Eva Kathrine Petersen
22. Johan P. Hansen
23. Thomas Kjellberg Christensen
24. Gorm Salomonsen
25. Jakob Grove
26. Nina Bargisen
27. Jacob Schach Møller

