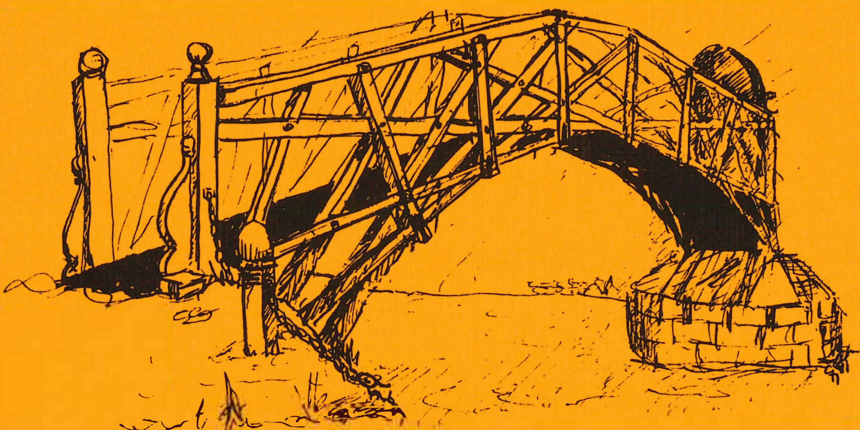
The Study Tour
by
Eulers Venner
to
England, October 1990



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by
Eulers Venner
Institute of Mathematics
University of Aarhus
Denmark
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Preface

This is the report of the study trip to England arranged by the mathematical association "Eulers Venner" ("Friends of Euler") at the University of Aarhus in Denmark. The trip took place 16-27 October 1990, with 31 students and 1 lecturer as participants. The association was founded 2 years ago following the first study trip for students at the Mathematics Institute in Aarhus, which went to Germany and Austria. In 1989 the association arranged a 3 days study trip to Copenhagen, and after the success of our latest trip to England, we hope it will be possible to arrange study trips abroad every second year in the future. Beside the study trips "Eulers Venner" arranges talks about mathematical subjects and is also responsible for different kinds of social get-together at the institute, e.g. a biannual celebration of the members that have graduated with a M.Sc.-degree.

The aim of the study trip was to learn about how mathematics is practiced in England in research, education and application. We hoped to get an impression of the English research and educational system by visiting different universities to see what kind of research is being done and what the conditions are like for students, both with regard to study and daily life (i.e. financial and social conditions). By visiting companies which apply mathematics and have mathematicians employed, we hoped to see examples of what kind of jobs a mathematician can have in industry, and what they are doing in such a job. With a visit to the Open University we hoped to get an impression of what university studies by correspondence involves. Finally we wanted to visit the British Museum, which has some of the most important sources for determining the state of mathematics in antiquity (papyrus Rhind and Babylonian cuneiform tablets).

In addition to the above there is also always a social reason for arranging a study trip. It strengthens the social ties between students at the Mathematical Institute.

The preparation for the trip started in late 1989. By asking staff-members at the University of Aarhus we obtained contacts at potential places to visit. We would especially like to thank Dr. Andrew du Plessis for helping us at this stage.

During the preparation we wrote to 140 trusts and companies for financial support. The following,

P. R. Electronics
Hewlett-Packard
Svend Bundgaards Fond
Marie og M. B. Richters Fond
Tuborg Fondet
Thomas B. Thriges Fond
Det naturvidenskabelige Fakultet, Århus Universitet

have, along with a private sponsor, supported our study trip financially. We take this opportunity to thank them very much for the financial support we have received from them, which made the trip possible.

We wrote to several companies and universities in England and got a positive response from: Warwick University, Oxford University, Thames Polytechnic, Hewlett-Packard, Cambridge University and the Open University. The study trip consisted of visits to these places, along with a visit to the British Museum. The trip turned out to be a success, both with regard to fulfilling its aim and giving the participants a good social experience, which brought them closer together.

We would like to thank the following people very much for making it possible for us to visit their institution/company and arranging the visit:

Dr. Ian Stewart (Warwick University),
Dr. Alan B. Tayler (Oxford University),
Dr. Mike C. Bramwell and Dr. Mayur K. Patel (Thames Polytechnic),
Catherine Slater and Roz Rowe (Hewlett-Packard Labs),
Dr. George A. Reid (St. John's College, Cambridge),
Dr. Jeremy J. Gray (The Open University, Milton Keynes).

We also want to thank everybody else who has been involved in making arrangements during our trip. In particular we want to thank Hans Hvas and Vivi Carstensen (former students from the University of Aarhus) for helping to make our stay in Oxford a pleasant one.

Henrik Vosegaard   Bernd Rühlicke   Kurt Ramskov

Participants

Anita Rohde	Jørn Flor Andersen
Bernd Rühlicke	Kim Rasmussen
Claus Jønge Jungersen	Kurt Ramskov Laursen
David Adams	Lars Bærentzen
Erik Jacobsen	Lars Hesselholt
Finn Søholm Larsen	Lars Kjær Larsen
Georg Larsen	Lars Thams
Henrik Just	Niels Horsbøl Eriksen
Henrik Vosegaard	Nina Bargisen
Ingelise Jensen	Peter Beier Gothen
Jakob Grove	Signe Skovmand
Jacob Schack Møller	Simon Daniel Kjeldahl
Jan Paradowski	Søren Munk
Jesper Kornerup	Tage Bai Andersen
Jesper Villadsen	Thomas K. Christensen
Jytte Madsen	Torben Bach Pedersen

Figure 1: All of us in the Council Chambers in Cambridge.

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Warwick University

The program for the trip included visits to several universities, the first of which was the University of Warwick, which we visited on October 18, 1990. We were looking forward to seeing a relatively new university and we expected it to be different, and perhaps less traditional, than the universities of Oxford and Cambridge.

Early in the morning we set out by bus for the university, which is situated at the boundary of the county of Warwickshire and city of Coventry, in the south of England. Unfortunately, we had difficulties finding our way and we arrived a little late at the Department of Mathematics at the university; the programme was therefore as follows:

11.30	Talk by Ian Stewart
12.30	Lunch
2.00	Nonlinear systems lab.
3.00	Free time

The first lecture was given by Ian Stewart, who was also organizing our visit. He gave introduction to the history of Warwick University in general and to the Department of Mathematics in particular, emphasizing the flexibility and the opportunities for students to combine subjects.

It was a very interesting lecture, and we have managed to obtain some booklets containing further information, which we have included in the following summary:

The University of Warwick is at present celebrating its twenty-fifth anniversary. It is characterized by modern architecture and by the fact that the Warwick campus is completely self-contained, which is a very significant feature from the point of view of many of the students.

The Department of Mathematics was one of the first departments of the university; it has been situated in various places on the campus and has grown in size; it has at present a total of 450-500 undergraduate students.

Mathematics at Warwick is organized according to the international pattern, which is unusual in British Universities. Both pure and applied mathematics exist within a single basic framework and are organized by

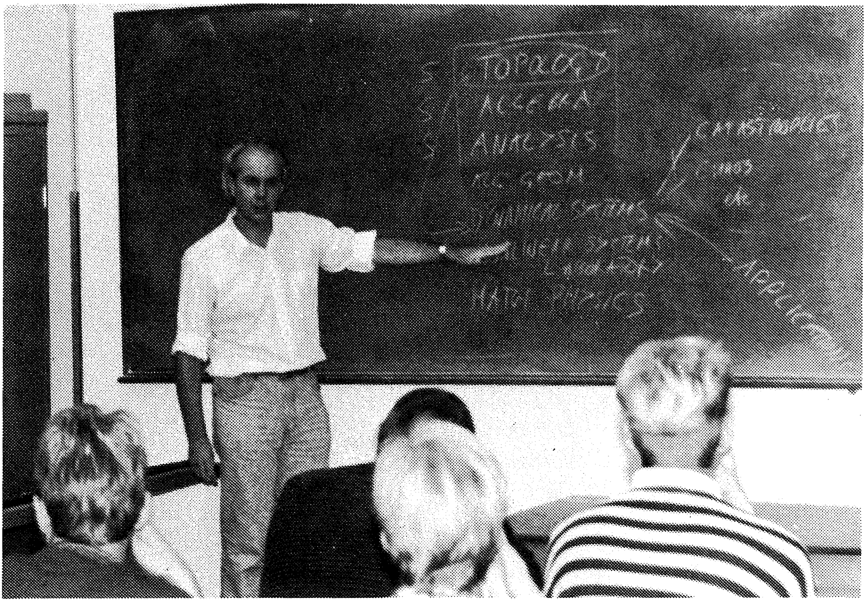


Figure 2: Ian Stewart.

a single department, whose name reflects this lack of distinction: Mathematics. In addition, many other types of applied mathematics are available as options, taught by people who work in the area of application. Thus there is a thriving group of mathematical sciences whose courses interact with each other to provide a thorough grounding in modern pure and applied mathematics in the broadest sense, which is illustrated in the diagram in figure 3.

Many of the courses for undergraduate students have been specially designed to allow combinations of subjects in such a way, that the degrees contain a core of mathematics together with a wide variety of courses:

1st year:	70 % core math.
	30 % options
2nd year:	50 % core math.
	50 % options
3rd year:	very free

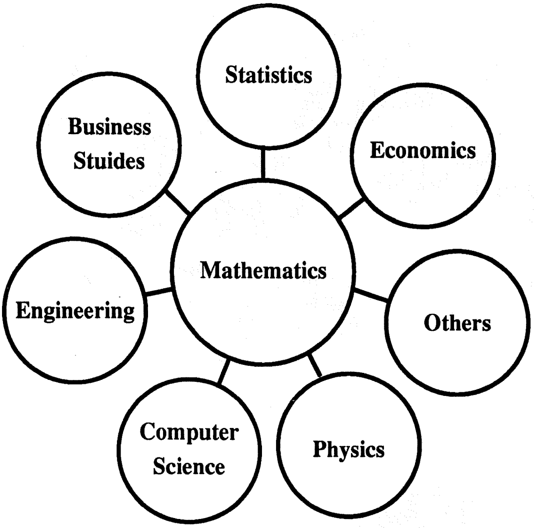


Figure 3: Subject interaction.

Warwick is rated by the Universities Funding Council as one of Britain's top five mathematics departments. It is highly active in research, and runs the Mathematical Research Center, and the Nonlinear Systems Laboratory. It has it's own experimental and computer laboratories. It has strong groups in the central areas of pure mathematics (algebra, analysis, and topology), in modern applied mathematics, and in the areas of mathematical physics, and it is setting up an interdisciplinary research programme in applied mathematics. It has well developed and longstanding international links, and each year attracts large numbers of the world's leading mathematicians as visitors.

The next point on the programme was lunch, and afterwards we had the opportunity to visit the bookshop and have a look around the campus.

Nonlinear Systems Lab (NSL)

After lunch we visited the Nonlinear Systems Lab. This person in the picture below, Greg King, appeared after a short while from behind a door marked NSL. He expressed some surprise at finding 31 people awaiting

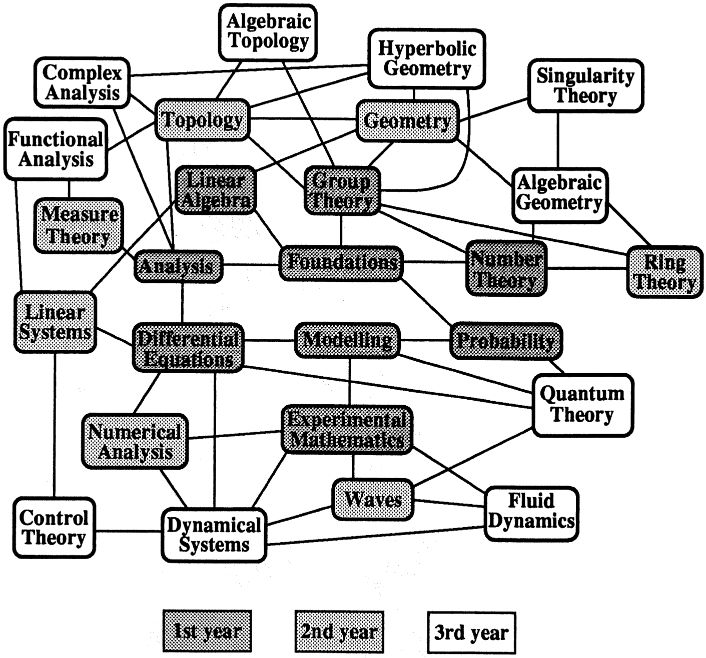
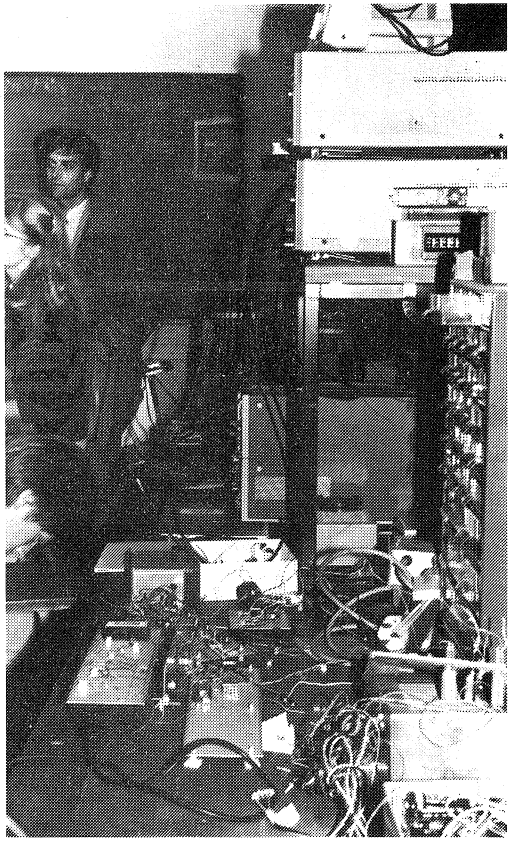


Figure 4: Schematic of selections from course structure. Mathematics is a network of interconnecting theories and methods, not an isolated collection of specialist topics.

his guidance instead of the 2-3 people he had expected. The lab itself looks quite similar to the labs used by physics students during their first three years of study at Aarhus. Despite the lack of leg space for 31 people and a slightly bewildered Greg King, we managed to squeeze together so much that the door could be closed and we were all able to watch the two PC monitors.

Dr. King started by explaining what was happening on the screen. The screen showed two circles, representing two cylinders, the smaller contained inside the larger, with their centers displaced. A computer programme simulated the flow of a fluid between the two cylinders when either one of the two cylinders were rotating with small angular velocity. He began with rotation of the inner cylinder. With the mouse he defined a point in the fluid and the program generated the trajectory of the point through the fluid. The trajectories divided into two groups.



Not surprisingly, points chosen near the surface of the inner cylinder moved around it. Points chosen on the other side of the center of the outer cylinder moved around in closed loops not containing the inner cylinder. This defined a partition of the fluid into to areas that did not mix. Then he conditions where changed so that the outer cylinder rotated slowly while the inner stood still. This again divided the fluid into two areas: an area with points whose trajectories formed closed loops which contained the inner cylinder, and another with points whose trajectories did not have this characteristic. These again did not mix.

We were told that this partitioning of the fluid cannot be avoided no matter how one mixes, as long as it is done slowly. This is in fact a problem in industry. What we where shown here was a two-dimensional problem simulated by a computer. Dr. King told us that they were in fact carrying out a real experiment with two cylinders but things were not working out so well on the day of our visit, so there was nothing to see.

Patterned chaos

From "Symmetric Chaos" by Ian Stewart and Greg King.

This is a description of an experiment with two concentric cylinders containing fluid as described earlier (the simulation on the PC was with non-concentric cylinders).

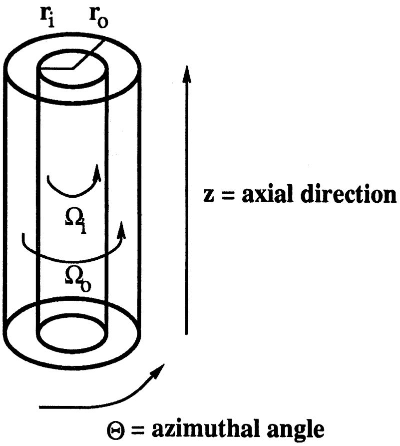


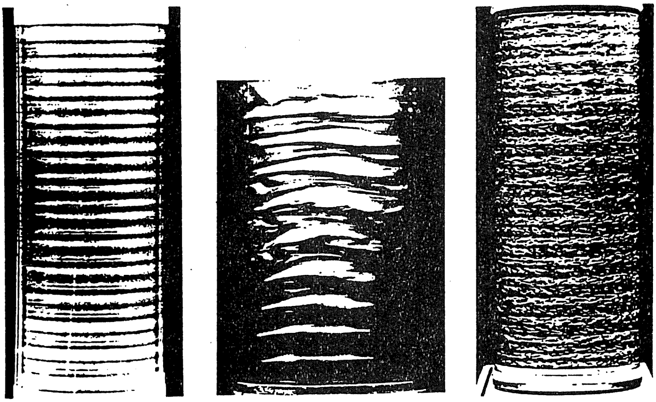
Figure 5: The Taylor-Couette apparatus. Here Ω_i, Ω_o denote the angular velocities of the inner and outer cylinders respectively, and r_i, r_o denote their radii.

Figure 5 shows the apparatus and figure 6 the result for different angular velocities. By fixing the outer cylinder and slowly speeding up the inner cylinder one obtains the following results as the speed increases.

1. *Couette flow*
   Laminar flow depending only on the radial position. Featureless.

2. *Taylor vortices*
   The flow stratifies into horizontal vortex layers, with the axial velocity components in neighbouring vortices acting in opposite directions.

3. *Wavy vortices*
   The interface between neighbouring vortices develops a wavelike structure, which travels azimuthally without changing form.



(a) (b) (c)

Figure 6: (a) Taylor vortices. (b) Modulated wavy vortices (near ends) and wavy turbulence (middle). (c) Turbulent Taylor vortices (b). [Pictures (a) and (c) courtesy of Harry Swinney and Randy Tagg.]

4. *Modulated wavy vortices*
   The wavy interface begins to oscillate axially as well as rotating azimuthally.

5. *Wavy turbulence*
   The wavy interface becomes even more complicated and turbulence sets in.

6. *Turbulent Taylor vortices*
   The stratified pattern of Taylor vortices reasserts itself, but now the overall flow is turbulent and the interface between vortices is 'flat' only 'on the average', in the following sense. If the velocity of the flow is averaged over a period of time then along the 'boundary' between turbulent vortices, the average axial velocity is zero. The comparison between Taylor vortices and turbulent vortices is shown in figure 2.

Coupled oscillators

After the demonstration of the computer program on the PC's, Dr. King showed us an oscilloscope which was portraying the phase diagram of two identical coupled oscillators. After fixing the frequency so that it was identical for the two oscillators, the remaining variables was the voltages across the two oscillators. The coupling had two attractors, wells. It is possible to visualize the coupling as a particle in two potential wells, characterized by the voltages. The particle moves from well to well in a trajectory, the phase diagram, and exhibits quite chaotic behavior. By changing the depth of the wells, it is possible to catch the particle in one of the wells as Dr. King showed us. After this he demonstrated period doubling in the coupling by raising the voltage slowly. This is analogous to the period doubling in a Feigenbaum system:

$$X_{n+1} = CX_n(1 - X_n)$$

where the variable C is determined by the voltages across the coupling.

Conclusion

We were pleased to visit Warwick University and hear about a degree structure which is obviously more flexible and contains more possibilities for combining subjects than our own. We also found it very interesting to see the Nonlinear Systems lab, the equivalent of which does not exist in Aarhus.

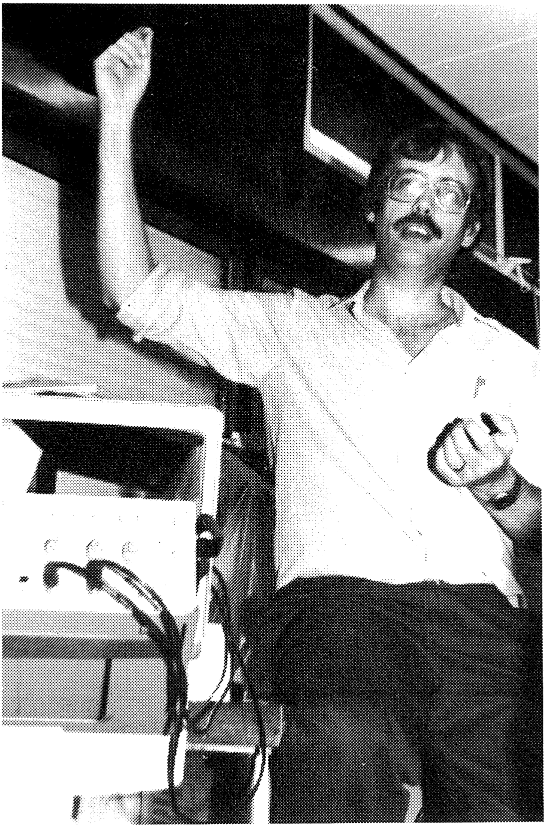


Figure 7: Greg King.

Oxford University

On Friday October 20, 1990 we visited the mathematics department at Oxford University. We arrived at 10 o'clock and were welcomed by Dr. Alan B. Taylor, a mathematician at the department of applied mathematics, who was organizing our visit. The programme for the day was as follows:

10 am	Introduction by Dr. Allan B. Taylor.
11 am	Coffee-break in the graduate students' meeting room
2.30 pm	Lecture by Dr. Roger Penrose
4 pm	Lecture by Dr. E. H. Kronheimer

Introduction by Dr. Alan B. Taylor

Dr. Taylor started by giving us an introduction to Oxford University in general and the Faculty of Mathematical Sciences in particular, and went on to describe the educational system at Oxford, a system with long traditions. Finally he told us about his own area of work, applied mathematics, and what was going on in this area at Oxford.

The University of Oxford is rich in traditions, due to its long and eventful history. It was founded 800 years ago, making it the oldest university in Britain. It began by students meeting to discuss religion and philosophy – mathematics did not interest them at this time.

The students gathered into groups, and this resulted in the foundations of colleges. Some of the first colleges were University College (1249), Balloil College (1260), Merton College (1264) and Exeter College (1314). For a long time only men were allowed in the University. The first college for women was Lady Margeret Hall, founded in 1878. One of the most recent colleges is St. Catherines College (1966), which we mention for 2 reasons. Firstly, it was designed by a Danish architect, Arne Jacobsen. Secondly (and more important) a former student from Aarhus, Hans Hvas, who is currently a student there, was able to take some of us to dinner there, so we were able to experience the special atmosphere

Figure 8: Outside the Mathematical Institute in Oxford.

of an Oxford College at first hand. (Coming from a relatively non-class-conscious society it made a big impression on us to see undergraduates sitting on benches, while post graduates sat on cushioned chairs and lecturers sat elevated over the rest at the *high table*.)

In the beginning the colleges were separate institutions but nowadays the teaching and examining is done centrally, although the colleges are still responsible for their own economies. Admission fees are about £3000-3500 pr. year depending on the college, although when boarding fees, cost of books, etc. are taken into account the minimum cost is about £7500 pr year.

An interesting historical detail is that until the 1930's all students had to wear a gown. The reason was that students were liable to be attacked (!) by people from the town, and by wearing a gown students could identify one another and come to each others assistance.

The Faculty of Mathematical Sciences has at present 600 undergradu-

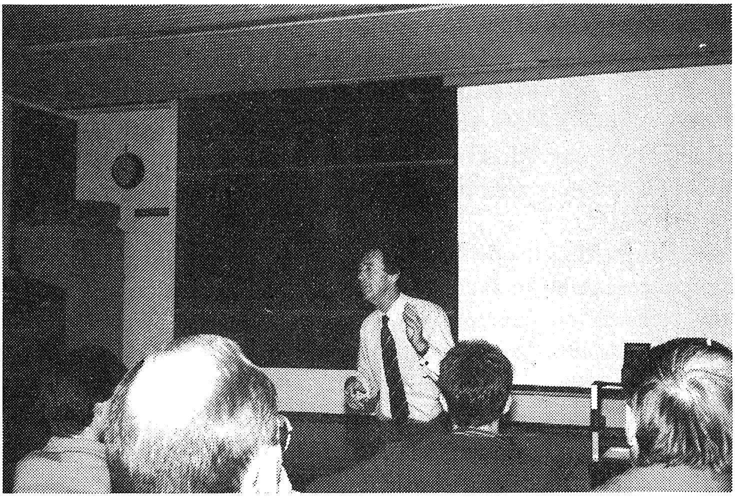


Figure 9: Dr. Allan B. Taylor.

ate students (200 pr. year). These are students studying for the Bachelor degree. There are 150 post graduate students, comprising of 80 studying for the 1-year Master of Science degree (M.Sc.) and 70 studying for the 2-year Doctor of Philosophy degree (Ph.D.). The research staff consists of 20 people doing post doctoral research, without tenure (i.e. employed on a temporary contract), 80 faculty members with tenure and 7-8 full professors. The faculty is run exclusively by the professors, in sharp contrast to our university where lecturers, students and administrative personnel all have a say in the running of the university.

The faculty is divided into departments, some of which are computer science, statistics, philosophy of mathematics, applied mathematics and pure mathematics. When enrolling students have to choose one of these departments – it is not possible to be enrolled in more than one. However, students in a particular department are able to follow some courses offered by other departments.

The educational system is as follows: to become enrolled in the university students have to be admitted to one of the colleges. Each student is assigned a tutor at their college, who advises and supervises the student on all aspects of his/her studies. Tutors are always members of the university staff – not older students as in Denmark. Lectures and examinations are given centrally by the relevant department of the university, but exercise classes and the like are held at the colleges in small groups of about 8 people. Most university staff, Dr. Taylor included, have the double role of being a lecturer at the university and a tutor at one of the colleges.

Despite its long traditions the university is not old-fashioned when it comes to research. In fact, 20 years ago the applied maths department began working on concrete problems in industry. People in industry were invited to come and spend a week at the university during which they discussed their problems with the mathematicians, who reformulated the problems in a mathematical way and worked on solving them. This interplay between mathematicians and people in "the real world" turned out to be very successful, both in solving industrial problems and in providing mathematicians with new ideas and inspiration in their own work. Dr. Taylor described some specific problems which they had been confronted with, including the problem of cliff-blasting: raw material is obtained from a cliff-side by blowing up part of it.

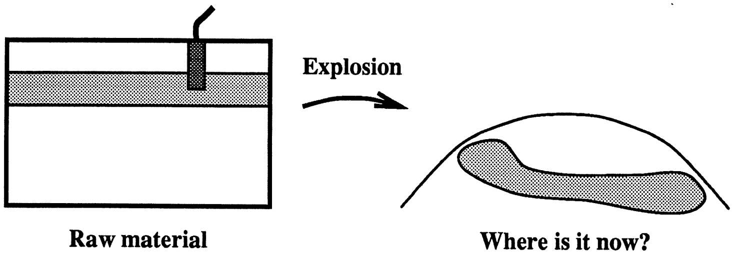
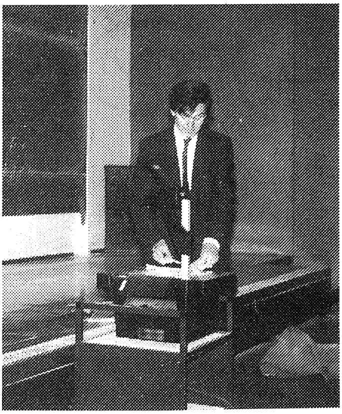


Figure 10: Before and after the explosion.

The problem is to find out how the raw material is distributed in the rubble after the explosion, so that it can be extracted with greatest efficiency.

After Dr. Taylors talk we had a coffee-break in the graduate students' meeting room, where we were able to meet some of the maths students and hear about what studying in Oxford was like from a students point of view.

Dr. Roger Penrose: Quasi-crystals



Whereas proper crystals only have 1-, 2-, 3-, 4-, and 6-fold rotational symmetry, it is possible to have lattices which deviate infinitessimally from being crystals and which have n-fold rotational symmetry from random $n \in \mathbb{N}$.

These are called quasi-crystals, and were the subject of Dr. Penrose's talk. He showed us a specific example of how a quasi-crystal could be built up starting with a few simple figures. The quasi-crystals Dr. Penrose talked about were all 2-dimensional, but he remarked that the situation was similar in higher dimensions. He finished by showing us examples of substances in nature which appear to have a quasi-crystal structure.

This was an interesting, impressive and entertaining lecture which we cannot give full credit in a short resume. For further details of part of the lecture we refer the reader to Dr. Penrose's article "Escher and the Visual Representation of Mathematical Ideas" in "M. C. Escher: Art and Science", edited by Coxeter, Penrose et al. (North Holland, 1987).

Dr. Penrose mentioned two articles with more information: In *Scientific American*, January 1977, Martin Gardner writes about nonperiodic tilings, and in *American Scientist*, November-December 1986, Paul Joseph Steinhardt has an article titled "Quasicrystals" (this article also has many references).

Dr. E. H. Kronheimer: 2- and 4-dimensional manifolds

The talk by Dr. Kronheimer was an outline of some of the problems and results in the classification of manifolds. Of special interest are recent results (by the Oxford mathematicians S. Donaldson, M. Freedman and others) to the effect that the 4–dimensional case is very different from other dimensions.

The first part of the talk introduced surfaces in 3–space, familiar examples of which are shown in figure 11.

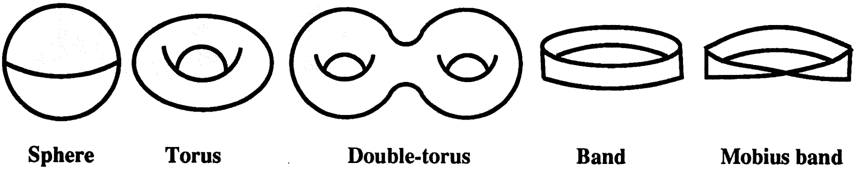


Figure 11: Surfaces in 3–space.

These are 2–dimensional manifolds, that is, they can locally be described using 2 coordinates. One way to construct such spaces is to obtain them by making identifications on simpler spaces. E.g. the Möbius band might be constructed by identifying (gluing together) opposite edges in a square; the torus is the result of the construction in figure 12.

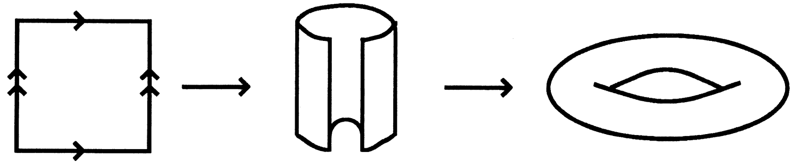


Figure 12: Construction of the torus.

This method also gives rise to surfaces not embeddable in 3–space; examples are the Klein bottle and the projective plane (figure 13). The previous, like the Möbius band, are non-orientable surfaces (i.e. there is no way to distinguish two sides).

3–dimensional manifolds can be obtained by similar constructions E.g. the 3–torus is a cube with opposite sides identified. Slightly more complicated is the dodecahedral space of Poincaré: Identify opposite sides in a dodecahedron with a rotation of 72° (figure 14).

The question arises as to whether it is possible to give a complete classification of all manifolds of any given dimension. The answer turns out to depend strongly on the considered dimension. We consider only *closed*, smooth manifolds, i.e. compact manifolds with no edges. The desired classification should only consider as separate those manifolds that

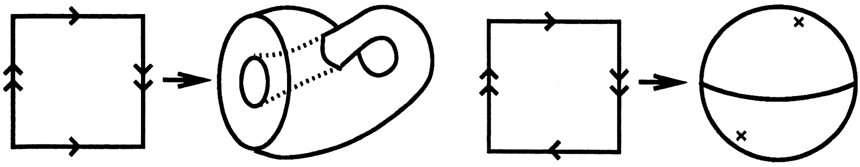


Figure 13: Construction of the Klein bottle and the projective plane.

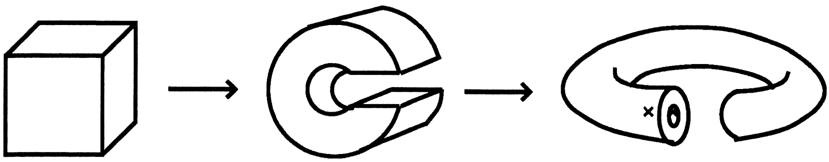


Figure 14: Construction of the dodecahedron.

are *topologically* different. One of two viewpoints might be taken: manifolds should be identified when they are homeomorphic (i.e. there exists a *continuous* mapping with *continuous* inverse between them) or manifolds should be identified when they are diffeomorphic (i.e. there exists a *smooth* mapping with *smooth* inverse between them). That these two concepts are not always the same was illustrated by J. Milnor in 1956, when he showed that there existed 28 different manifolds all homeomorphic, but not diffeomorphic, to the 7–sphere.

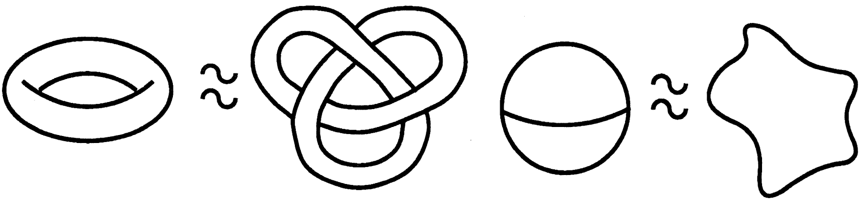


Figure 15: Homeomorphic manifolds.

Dr. Kronheimer gave a short historical overview of the solving of these classification problems and the men behind it. It turns out that dimensions higher than 2 remained unsolved for a long period of time.

- dim 1: Is more or less trivial: It is not difficult to show that the circle is essentially the only 1–dimensional closed manifold.
- dim 2: A "classical" result in topology.
- dim 3: Partly solved in the 1970's and 80's by Thurston and Gromov using hyperbolic geometry.
- dim 4: Nobody knows anything until 1982.
- dim>4: Success in the 1960's by S. Smale, J. Milnor, Wall, Kirby and others. Unfortunately the methods used don't apply to lower dimensions.

The classification of 2–manifolds is simple to describe: There is a series of *orientable* surfaces and a series of *non-orientable* surfaces of which the latter cannot be embedded in 3–space. The orientable surfaces are illustrated in figure 16.

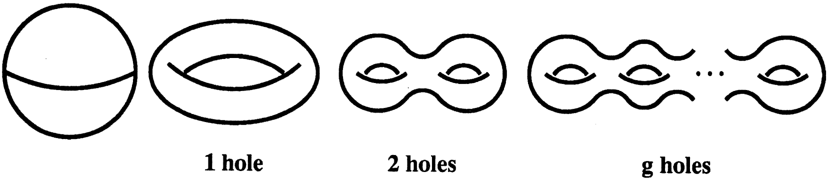


Figure 16: Orientable surfaces.

The non-orientable surfaces are obtained from the sphere by cutting out a number of discs and gluing the edge of a Möbius band (which is a circle!) to the edge of each of the holes. One hole gives the projective plane, two holes give the Klein bottle (figure 17).

This complete and simple list can be described via algebraic topology. I.e. it is possible to attach to the manifolds algebraic structures (homology and cohomology modules) that capture all the geometric information, so that two closed smooth 2–manifolds are homeomorphic or diffeomorphic (which makes no difference in this case) if and only if the algebraic structures are isomorphic. This is however *not* the case in dimension four (four is a more "geometric dimension"), so other methods must be applied.

In 1982 two things happened:

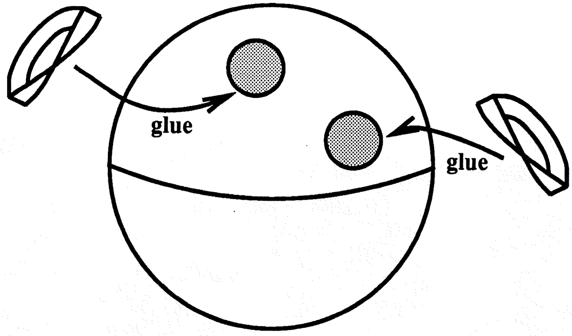


Figure 17: Construction of Klein bottle.

1. M. Freedman showed that with respect to the homeomorphic viewpoint dimension 4 is no different from other dimensions; in fact, the problem is the same as in dimensions 8, 12, etc.

2. S. Donaldson showed that with the diffeomorphic viewpoint dimension 4 is *very* different from other dimensions.

Donaldson's results are inspired by the so-called Yang-Mills equations from theoretical physics, which play a role in the theory of weak and strong interactions; analogous to the Maxwell equations in electromagnetism. The Yang-Mills equations makes sense in any dimension, but comes into their own only in dimension 4, because of certain symmetries. Donaldson studied these on "time-space spaces". According to Dr. Kronheimer the results have, however, not (yet?) had any influence back into physics; which from an intuitive viewpoint could seem likely, because of the role of 4 dimensions in physics (time+space=4 dim).

Conclusion

It was interesting to see that the Mathematical Institute had a department for applied mathematics with contacts to the outside world – unlike in Aarhus. The last two lectures showed us however that pure mathematics has an extremely high level. Even if Oxford University has old traditions, it does not seem oldfashioned.

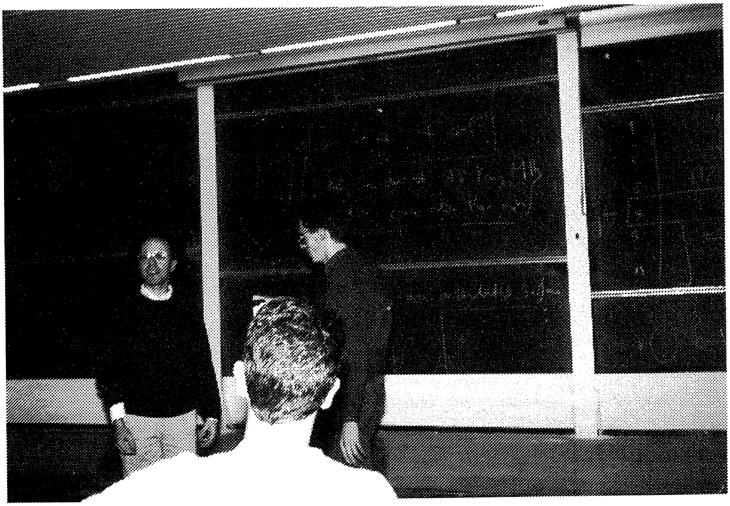


Figure 18: Dr. E. H. Kronheimer.

British Museum

On Sunday October 21, 1990 we visited the British Museum in London. One of the participants, Kurt Ramskov, had prepared a 'guided tour': "A glimpse of the Mathematics of ancient Egypt and Babylonia", lasting about one hour and showing items especially interesting from a mathematical point of view.

Egypt and the Hieroglyphs

The reasons we know so much about ancient Egypt are

1. the climate is very dry, so things are better preserved,
2. the ancient egyptians were very industrious when it came to writing things down (nearly all aspects of Egyptian life were recorded) and
3. a lot has been written on hard materials such as stone, and thereby preserved for a very long time.

The tour started in front of a part of the wall of a temple on Abydos, containing a list of the kings from a particular period of Egypts history. It is sources like this which have made it possible to obtain so much knowledge about the history of ancient Egypt.

It is usual to divide the history of ancient Egypt in the following periods in figure 19.

The old Kingdom was the period in which the pyramids were built, the Middle Kingdom was the classical period in which art flourished and the New Kingdom was the period in which Egypt was a military superpower and ruled the neighbouring countries. It was during this period that the tombs in the valley of Kings were built.

A prerequisite to determining the history of ancient Egypt is being able to decipher the old Egyptian script, and this led us to the Rosetta-stone, one of the most famous items at the British Museum. The Egyptian letters are called hieroglyphs, which means secret sign (today the word is also used in everyday language for something impossible to read!). The most recent known hieroglyphic inscription is from 400 AD. For the

Prehistoric time	:	until 3200 BC
Archaic time	:	3200-2700 BC
Old Kingdom	:	2700-2250 BC
The first interval period	:	2250-2150 BC
Middle Kingdom	:	2150-1780 BC
Hyksos period	:	1780-1580 BC
New Kingdom	:	1580-1080 BC
The third interval period	:	1080-750 BC
Sais period	:	750-525 BC
Conquered by the persians	:	525-323 BC

Figure 19: History of ancient Egypt

next 1400 years no one was able to read hieroglyphs. In 1799 the emperor Napoleon decided to attack Egypt, which was then part of the British empire. He sent a group of scientists to Egypt who collected many historical items. This was the beginning of egyptology – the science of ancient Egypt. While digging trenches to fend off the English army a French soldier found the Rosetta-stone close to the town Rosetta, in the delta of the Nile. Later the French were beaten by the English and all the collected items were then sent to the British Museum.

The Rosetta-stone contains inscriptions in Hieroglyphic, Demotic and Greek, and the stone has been the major source which made it possible to decipher the hieroglyphs. The greek inscription could be read immediately after the finding of the stone. The text states the right of the temples to have their own taxes, and a promise from the king not to increase the taxes to the crown. It was written in the time of Ptolemy 5. in 196 BC, and it was written in the 3 languages because the present rulers were Greeks, Hieroglyphic was the written language of the priests and Demotic was the written language of the people.

It was the French egyptologist Champollion (1790-1832) who first succeeded in deciphering the hieroglyphs. The major source of inspiration was the Rosetta-stone. Prior to this the Dane Georg Zoëga had noticed that some hieroglyphs are contained in a oval frame (called a *cartouche*). They turned out to be the names of kings or queens, which was the first step in breaking the 'code'. The difficulties in deciphering the hieroglyphs are due to the fact that hieroglyphs are not letters. The same hieroglyph can be a word, a syllable or a letter depending on the context. As with the Semitic scripts the vowels are omitted in Hieroglyphic, and normally it has to be read from right to left. However, on monuments the hieroglyphs are sometimes written from left to right, or vertically.

The hieroglyphs are the equivalent of printed in the present day. Hieratic script - which is the usual on papyri - is the equivalent of handwritten text in the present day.

Passing through several rooms with items from ancient Egypt, including a part of a colossal statue of Ramesses the Second, we arrived at the mathematical papyrus Rhind.

Papyrus Rhind and the Egyptian Mathematics

The papyrus was bought by A. H. Rhind in 1857 in the town Luxor in Egypt. It consists of two major pieces, but only one of them was exhibited. Papyrus Rhind is together with the smaller papyrus Moscow the only knowledge we have of mathematics in ancient Egypt. On the other hand they are original sources (no original sources exist for the mathematics of ancient Greece for example). Papyrus Rhind was written in 1575 BC at the end of the Hyksos period, but the author Ahmose writes on the papyrus that it is a copy of papyrus from the 12th dynasty, around 1850 BC.

The mathematics of ancient Egypt is applied mathematics. It involves calculation of areas, distribution of bread to workmen, food to animals etc. Nowhere is there any evidence of arguments or proofs. Everything looks as if it is knowledge gained through experience.

The numbers were written in a decimal system (see figure 20). That made addition and subtraction quite simple. Multiplication was done by the doubling method (multiplication of 18 by 13 is demonstrated in figure 21). In a similar fashion division was done by halving.

The Egyptians were also able to calculate with fractions, but they only wrote fractions of the form $1/n$ (written as the symbol for n with a dot above). When multiplying fractions by the doubling method they needed to be able to write $2/n$ as a sum of fractions of the form $1/k$. The first part of the papyrus Rhind, which was not exhibited, contains a table of this for n running through all odd numbers from 5 to 101.

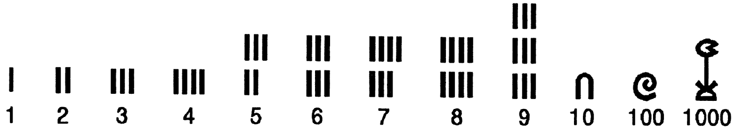


Figure 20: The hieroglyphics for numbers.

	1	13
x	2	26
	4	52
	8	104
x	16	208
	18	234

Figure 21: Multiplication of 18 by 13 in the Egyptian way.

The part of the papyrus Rhind exhibited contains a lot of exercises. There was a group of geometrical exercises, where drawings of circles, rectangles etc. could be seen on the papyrus. Kurt had brought a Danish translation of some of the geometrical exercises so we could 'read' the hieroglyphic exercises.

Babylonian Mathematics

After seeing the Egyptian collection we proceeded to the Babylonian room. Mesopotamia, the region between Euphrates and Tigris, does not have as orderly a history as Egypt. Wars and invasions have occurred very frequently in this region, and the situation is no different in the present day as most of the region is now included in Iraq.

The climate is not as ideal for preserving things as the Egyptian, but despite this we know a lot about the history of Mesopotamia because they wrote in clay. Once the clay tablets dry they are not affected by the climate. The written language used in Mesopotamia was the cuneiform script. It was used until around year 0 BC. A lot of different languages were written in the cuneiform script, including old-Persian, and it was an old-Persian inscription at Persepolis which was the first Cuneiform inscription to be deciphered, by the German Grotefend in 1802. Again it was the names of the kings which was the key to deciphering. Today most of the cuneiform tablets can be read, and this has given a lot of information about Mesopotamia's history, because archaeologists have found libraries with thousands of tablets. Most of them are either letters, or about matters concerning administration, but also more exciting things have been found including tablets containing a glossary between two languages, a tablet with the flood legend (quite similar to the one in the Bible) and mathematical texts. At the British Museum a collection of tablets with very different contents was exhibited.

Two cuneiform mathematical tablets were exhibited. One of them had a table of n and n^2 from which square roots could be extracted. The numbers were written in a position system with base 60, and knowing the symbols for 1 and 10 (the numbers from 1 to 59 in every position were written in a decimal system) it was even possible to read the table of squares.

The other tablet contained a lot of exercises including some ditch digging problems which some of us had met in primary school. The Babylonian mathematics is in many respects similar to the Egyptian. Formulas and methods are described, but arguments and proofs are never given. A lot of the the exercises are concerned with practical problems (e.g. ditch digging), but there are also exercises which could not have been related to practical situations showing that at least some people in Mesopotamia did mathematics for other reasons.

A lot of the exercises involve solving equations. The formula for the general quadratic equation was known.

Old Watches and Dice

The tour was now finished, but those who were interested followed Kurt from the Babylonian room to a room with old clocks and watches. We passed an exhibition of old dice, astragalis etc. (playing items from the greek period and earlier) on our way. Finally we went to the British Library, where there was an exhibition of old scientific writings: a letter from Newton to Hook and a book written by Leonardo da Vinci in mirrorscript. The rest of the afternoon was free to see the other items in the museum.

Conclusion

Not surprisingly the huge British Museum attracts a lot of visitors on a Sunday afternoon. The different groups of visitors and their guides were almost fighting to reach the best viewpoints to the famous historical findings. However, Kurt managed to ignore the masses to give us a profound description of the things we saw, and as we moved on from the exhibitions of common interest (e.g. the Rosetta-stone) to those of more specific mathematical interest, it became easier to be gathered as a group.

After the guided tour, people spread to investigate the museum on their own – a task, that is hardly finishable no matter how little archeology might mean to you.

Even if things are well presented, you cannot overview a museum that contains exhibitions of cultures and subjects from almost every corner of the history of the World on any limited period of time – at least not the British Museum on only one Sunday afternoon.

Thames Polytechnic

We visited Thames Polytechnic, School of Mathematics, Statistics and Computing on Monday October 22, 1990. The purpose for our visit to Thames Polytechnic was to hear about what kind of research work they were doing and what their educational system was like. The programme for the morning consisted of the following lectures by teachers and a postgraduate student on their recent work and studies:

- The role of mathematical modeling in industry and the environment - review of current research at Thames Polytechnic. By Dr. K. Pericleous
- An application of graph theory in the social sciences. By Professor M. G. Everett
- Parallel algorithms for some engineering problems. By Dr. C. H. Lai
- Design strategies for computational fluid dynamics software. By Mr. M. Petrides
- An intelligent decision support system for the control of the activated sludge process. By Dr. B. Knight

In the afternoon we heard about the different kind of educational possibilities there are at Thames Polytechnic, and which courses it is possible for a student to select, both as an undergraduate and as a postgraduate:

- Courses at undergraduate level at Thames Polytechnic. By Dr. F. M. Tyler
- Postgraduates at Thames Polytechnic. Mr. M. C. Bramwell

The day ended with a tour on the school, during which some students showed us projects they were working on in cooperation with companies.

Dr. K. Pericleous: The role of mathematical modeling in industry and the environment – review of current research at Thames Polytechnic

One of the chief interests of the department is mathematical modeling. That is, the process of making abstract models of reality. Usually the model is a set of equations.

Dr K. Pericleous told us about one area of mathematical modeling, namely computational fluid dynamics (CFD). Setting up a model in CFD involves specifying the geometry of the area in question – walls, entries, exits, etc. and a specification of the fluid – temperature, pressure, etc.

The CFD-predictions are then based on a set of values of the fluid variables which describe the state at some initial point in time. The predictions are rarely 100% realistic. This is due partly to the facts that they are based on measurements at a finite number of locations in space-time, and the input data is only approximate.

In these predictions more information is available than in experiments. Another quality of and reason for using CFD is that there are a lot of areas where experiments cannot be made; an extreme example is: determine the dependence on the weather of the spread of radioactive particles from a leak in a nuclear power plant.

Currently these techniques are used for modeling iron casting, the production of gases in turners, the air and heat flow in buildings and a wide variety of other fields.

Professor M. G. Everett: An application of graph theory in the social sciences

Professor Everett showed us an interesting mathematical problem arising from the application of graph theory to social science.

Suppose you have a connected graph without loops. Then call a colouring of the graph regular if whenever two nodes have the same colour they have the same colour set, where the colour set of a node is the set of colours of the neighbour-nodes. There are two trivial regular colourings: all nodes have the same colour, and all nodes have different colours. In the application the nodes are people and the edges their relations. In figure 22 is shown a graph where all regular colourings are trivial.

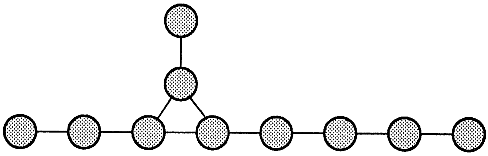


Figure 22: A graph with only trivial regular colourings.

Dr. C. H. Lai: Parallel algorithms for some engineering problems

This talk was about the rethinking of 'classic' algorithms in order to implement them effectively on a parallel computer. A number of examples were presented, and among those we have chosen to write about the "Gauss-Jordan elimination".

Given an $n \times n$ matrix A and an $n \times 1$ vector b, the problem is to solve $Ax = b$ for the unknown $n \times 1$ vector x. The Gauss-Jordan algorithm consists in the elimination of all unknowns but x_i from the i'th equation. The solution is then obtained directly.

Given a CREW SM SIMD (that is Concurrent Read Exclusive Write, Shared Memory, Single Instruction Multiple Data) Computer with n^2+n processors, the following algorithm will work (assuming that there is a unique solution, that `A(j,j)`$\neq$`0` and that b is the $n+1$'st column in `A`):

```
PROCEDURE Gauss-Jordan(A,b,x)

  1:  FOR j=1 TO n DO
        FOR i=1 TO n DO IN PARALLEL
          IF i<>j THEN
            r := A(i,j)/A(j,j)
            FOR k=j TO n+1 DO IN PARALLEL
              A(i,k):=A(i,k)-r*A(j,k)
            ENDFOR
          ENDIF
        ENDFOR
      ENDFOR
```

```
2:  FOR i=1 TO n DO IN PARALLEL
      x(i):=A(i,n+1)/A(i,i)
    ENDFOR
```

The time complexity is $O(n)$.

M. Petrides: Design strategies for computational fluid dynamics software

There is a growing demand for high-quality CFD-software, but what is software quality?

Software quality can be split into two major parts: External and internal quality.

External quality includes things like

> Efficiency, reliability, extensibility, reuseability, compatibility, portability (especially from 1-processor architecture to m-processor architecture), integrity and userfriendliness.

Internal quality is

> Modularity, cohesive software modules, low intermodular coupling (that is information hiding), structure, readability etc.

In order to achieve all of the above goals the following software design strategies have to be considered:

1. Dataflow-diagrams (the top-down approach).
2. Datastructure, entity-relations (mainly Jackson system development).
3. Object Oriented Programming (by popular request!).
4. Expertsystems (to capture the users tacit knowledge).

A 'blackboard' design for a CFD-prototype was presented, it tries to incorporate all of the ideas presented above. The 'blackboard' design will not be explained here, but it reflects a lot of the ideas in the 'DAT31' course at Aarhus University!

Dr. B. Knight: An intelligent decision support system for the control of the activated sludge process

Dr. B. Knight worked with a system which should be able to make intelligent decisions to control an activated sludge process. The system could among other things help and/or guide the user to make decisions in some possible critical situations that might arise. This was done with a data-dictionary which was implemented as a tree-structure, where the nodes denoted possible states of the process, and the path between a 'father'- and 'son'-node denoted the cause that had forced the system from the 'father'-state to the 'son'-state. The expert system then works its way 'down' the tree, starting at the node describing the original state of the process and finishing at the node denoting the present state, and thereby deduces the cause(s) for this state. Furthermore it suggests which actions to take on basis of this, to bring process back to a 'normal' state.

Afternoon Session

In the afternoon session Dr. F. M. Tyler and Mr. M. C. Bramwell talked about the British education system and about the system on Thames Polytechnic in specific.

Thames Polytechnic is a young institution compared to other universities etc. in Britain. They have been teaching mathematics since 1940. The courses at undergraduate level are as follows:

- A full-time basis degree course of three years, or
- a "sandwich" of four years (where the third year is in industry).

Courses during the first two years have the following features:

- Modeling and problem solving.
- Emphasis on communication.
- Project work.
- Research and consultancy input.

For the final year the students can choose among a broad variety of courses:

- Expert systems
- Image processing and pattern recognition
- Parallel computation
- Software engineering
- CFD with application
- Algorithms and software for scientific computing
- Codes, ciphers and networks
- Medical statistics
- Applied systems modeling
- Econometric methods and applications
- Mathematical programming and discrete event simulation.

In reply to the question "what kinds of jobs do students get after they leave TP" Dr. Tyler could give the following statistic:

- 51% are working with computing
- 25% are working with math modeling and computing
- 13% are working with statistics
- 7% have jobs not directly connected with the courses
- 4% are teaching.

After this Mr. Bramwell talked about the postgraduate programme at TP and a little about research at TP, both pure and applied. Among other things he gave definitions of research and consultancy:

> "Pure research is done to satisfy the curiosity of the researcher."
>
> "Applied research is made in the hope that the result has financial value too."
>
> "Consultancy is done on request."

The courses given at TP can lead to a M.Sc. (Master of Science) or a Pg.D. (Postgraduate Diploma). After graduation the studies lead to a Ph.D. or a M.Phil.

At TP the staff and postgraduate students are currently split up into the following groups:

Academic staff	35	including 3 readers and about 5 research fellows.
Support staff	10	half of them are graduates and some postgraduates.
Research students	16	
M.Sc. students	50	about 10 of which are actually at TP; the rest are doing research in companies.

TP has a very close cooperation with industry and the school gets funds for research amounting to about £1.500.000 a year. The overall organization of research is done by Mr. Bramwell. The research takes place inside the following groups:

1. Center for numerical modeling & process analysis.
2. Expert systems group.
3. Center for applied statistics & systems modeling.

The students do not receive grants automatically but can get money from the Trading Agency:

£1600 for home students
£4500 for overseas students,

so as in Denmark most of the students have a job along with their studies.

After these talks we had discussions with some of the postgraduate students, who were so kind as to tell us about the projects they were working on.

Conclusion

We were very happy for the warm welcome to Thames Polytechnic and for including everything we could want in the programme: technical talks from a variety of subjects, information about the educational system at TP, and the opportunity to talk to students.

Hewlett-Packard Laboratories

We visited HP-Lab's in Bristol on October 23, 1990. After a rather long bus-trip from London (because of which we had to get up rather early), we arrived at HP-Lab's at about 10.15 am. We were all equipped with visitor's badges with our names on them, so that we could be distinguished from industrial spies. After that we were welcomed by Catherine Slater, who was organizing our visit, who gave a general introduction to the day's programme. Unfortunately we were not able to see the laboratories themselves because of the size of our group (32 persons). Instead some of the people working in the laboratories came out and spoke to us.

Catherine Slater, who was formerly a linguist at Oxford University but now in charge of relations to universities at HP-Lab's, remained with us all day and ensured that everything went smoothly.

Before proceeding with the programme we had coffee at the Thames Coffee area. An account of the various talks and discussion is given below.

Mark Gasteen

The HP-Lab's employ more than 650 researchers and engineers all over the world. HP-Lab's consists of three centers, one of which, the Information Systems center (ISC), is located in Bristol. ISC consists of two laboratories: The Networks and Communications Lab's (NCL), which employs 65 people and the Information Management Lab's, which employs 103 people.

The purpose of the lab's is to perform long term research and development, as opposed to the very specific research aimed at short term goals, which goes on in the divisions.

Also, the lab's contribute to basic research in scientific areas of interest to HP, and maintain a good scientific reputation in these areas.

The activities of HP-Lab's are of course influenced by philosophies of what the relationship is between basic science, research and development (R&D) and the customer

First M. Gasteen explained the traditional view of this relationship as a "technology food-chain" (figure 23).

Figure 23: Technology food-chain.

The picture is that basic research results obtained at the universities gradually flow through the system. In this process ideas from basic science are transformed into new products of use to the customers.

Contrary to this traditional point of view is the "technology ecosystem", which is the point of view that governs the activities of HP-Lab's. It states that there is an interaction between all the above elements, and is seen in figure 24.

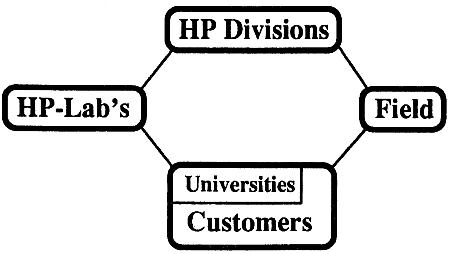


Figure 24: Technology eco-system.

A practical result of this point of view is that HP-Lab's cooperate with a number of universities throughout Europe. Among other things this cooperation involves offering students the possibility of staying at the HP-Lab's for a period of time.

The typical time scale for a R&D project at HP-Lab's is as follows: Initial ideas are developed in the lab's for about 2-3 years, and then in another couple of years the research should result in a product being manufactured. Altogether the time from idea to product is therefore about 5 years.

It was of course of particular interest to us to hear about the role of mathematicians in HP-Lab's. It turned out that about 10-15% of the employees actually were mathematicians. Their area of work was mainly within formal software engineering and security systems.

Overview of Information Management Lab

The overview was given by Phil Stenton, who started by telling us that he was a psychologist, and had been working for HP for 5 years.

The Information Management Laboratory (IML) is divided into 5 departments:

- Software Engineering
- Knowledge Based Programming
- Advanced Information Management
- Distributed Systems Engineering
- Distributed Office Systems

There is considerable overlapping between the last two departments.

One of the aims of IML is to give control back to the owner of the information, or put differently:

> "The structure of an information system should reflect the organization which owns and operates it, and not the other way around."

We heard more specifically about this problem when Phil Stenton talked about the Advanced Information Management department. Here the goal is to make users of a computer system able to work with it in their own professional language instead of forcing the users to learn the system-language in order to use the system properly.

One of the problems here is reducing the ambiguity in Natural Language Systems, which is what Phil Stenton is working on. We therefore heard some more about the ideas and experiments in this project.

Overview of Networks Lab by Pat Baker

The Network and Communication Laboratory works with the following topics:

- Instrumentation, which is what HP was making from the beginning.
- Network Management, which is about how you can be sure that a network is working correctly. This is clearly a very important field.

- Network Design, where the work is concerned about High Speed Networks, Distribution systems security and secure products in general. The security work is concerned mostly about communication systems, where it can be very important with good security (e.g. government networks).

The Lab is among other things right now working on a project about how you can expand local networks (e.g. a network on a university campus) to cover a whole country. This project works closely together with another project, which tries to connect networks in Atlanta, White Plain and San Jose – cities in the USA, which are placed quite far away from each other. This project is made on a "slow" network, which has a speed of transmission of 1.5 Mb/sec.

The lab is also working on making "fast speed" networks. That means a speed of transmission of 1 Gb/sec. It is often so that the same things are worked at at both speeds. The use of mathematics is quite difficult to point out, but it is mostly used to calculate whether a code (in an information transmission) can be broken or not, and to make statistical reports.

A question from the audience: "Is it mostly statistics that are made here, and is it an advantage to be a statistician to be employed as a mathematician here?" The answer was "no" to both things. Your special topic doesn't matter to an employment at HP. HP thinks that it's a problem to get people who are able to think, rather than to get people who are specialized in a specific topic. It's also more important that an employee has a general high education level and ability to think rather than knowledge of a specific topic, because the work changes all the time.

Roundtable discussion

The visit to HP was rounded off with a round table discussion. Besides Catherine Slater and ourselves, three young mathematicians employed by HP and two Danish scholar students participated in the discussion. As a prelude to the actual discussion, the three mathematicians, Waranda, Jonathan and Simon, spoke briefly on the subject: "Being a Mathematician at HP".

Waranda: To appease those of us with a craving for pure mathematics, Waranda started by listing the fields of mathematics, that she had been in touch with during the preceding six months:

- Category Theory
- Abstract Algebra
- Combinatorics
- Number Theory
- Probability
- Cryptography

plus some subjects saturated by the flavour of computer science. Thereafter she characterized her job as being divided into two main areas: Longer term projects and consultancy. The latter she thoroughly explained by four typical situations:

- People dropping by for a chat (very common).
- Explicit problems (seldom).
- "I'm sure you can help!"
- Participant in projects.

As an example of a longer term project, Jonathan presented his latest work:

Jonathan: The project consisted in determining whether information about procedure complexity decreased the number of faults caused by people moving around in computer networks. He designed an experiment, a mathematical model reflecting it and finally used statistics to find the answer: No!

But unlike university research this did not conclude his work: he had to explain his result and methods to people that, as he put it, aren't exactly dim (...). Afterwards it was decided that he should write a paper on his work. The fact that HP-Lab's publish (some of) their results surprised a lot of the participants on the trip.

Simon: The last of the mathematicians, Simon, started by claiming that he was the only real mathematician at the HP-Lab's in Bristol: he had a Ph.D. from Warwick on the geometry of supermanifolds (Waranda objected). He also explained that his reason for taking a job at HP-Lab's was the lack of job opportunities at the universities. Finally, he told a little about the project he was presently working on: Optical Character Recognition. His group used "computers" based on neural networks, as they are expected to be able to simulate the reading process better than ordinary digital computers. Actually some scientist are trying to make such computers simulate Darwinism. No wonder they have not succeeded yet!

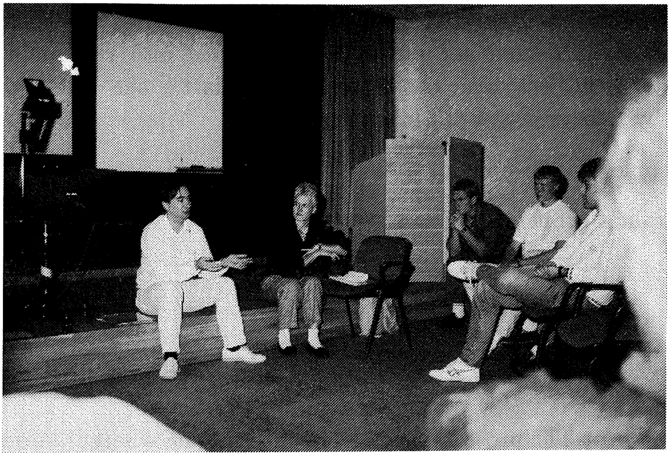


Figure 25: Simon and Catherine.

Discussion

Simon: Working in a private company requires a lot of patience when it comes to explaining your work to others, such as managers, engineers etc.

Catherine: It's necessary to tell others about your work – otherwise it's useless.

Simon: I experience mathematicians abilities to be complementary to the abilities of others. As a consequence we are very good at tracing other people's errors.

Jan: After what I have been told today, I believe our education is very well suited to the demands of HP-Lab's.

The discussion moved towards application of computers in mathematics.

Jonathan: The american HP are trying to factorize large numbers into primes.

Simon: In my ten percent free time I try to prove the Riemann hypothesis using computers.

Jonathan: I'm studying part-time for a Ph.D. in London and enjoy very much the access to the powerful machines here at HP.

Waranda: My Ph.D. was on Verma modules in characteristic p. I missed the contact to reality and wouldn't dream of doing that for the rest of my life!

The main subject was changed again when Tage raised the question:

Tage: How are decisions made as to whether to go on with a project or not, or whether something is applicable?

Simon: Risk-benefit!

Jonathan: You are not told to stop – you "agree" to stop.

Catherine: You have to convince others that what you are doing is any good.

Waranda: I was on this project, but we just didn't think that the market was ready for it yet. So we stopped it.

Finally there was some discussion on the working conditions at HP.

Catherine: You decide yourself when to begin and when to end your day. I believe it's the only way to make things work: It's results that count, not working hours.

Jonathan: I can't imagine working conditions to be more broad-minded than they are here. I think that whenever you're involved in a longer term research project this is a necessity. Just like the universities.

Conclusion

It was an altogether interesting visit, during which we felt very well looked after at all times. One of the most important aspects of it was the discussions between us and the HP-people. From this point of view the most interesting part of the arrangement was probably the roundtable discussion. Its success was due to the fact that we got to talk to other mathematicians working in the "real world", which of course was of major interest to us. It was also very interesting to learn about the management structure of HP-Lab's, which relies heavily on people's own initiative and ideas. In contrast to this the overviews of the specific computer scientific problems being worked on in the laboratories were of less interest, especially to those of us with a non-computer-science background.

In short it was a well-planned visit, during which we had the opportunity to have some very fruitful discussions on various topics.

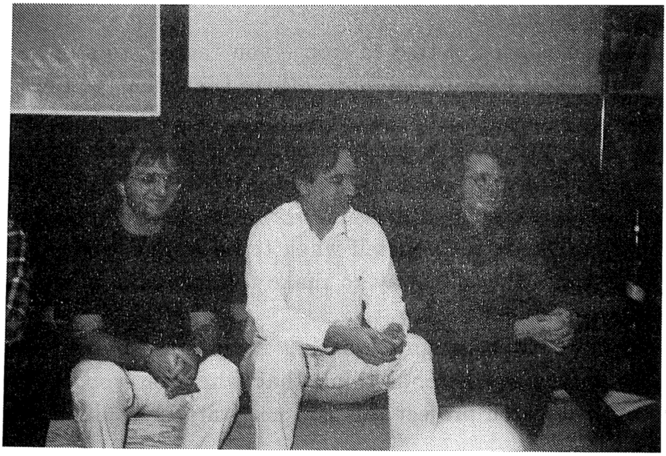


Figure 26: Jonathan, Simon and Waranda.

Cambridge University

On Wednesday October 24, 1990 we visited Cambridge, where we arrived at 10.45 after a $2\frac{1}{2}$ hour bus trip. At 11 o'clock we were welcomed at Guild Hall by the fully decorated mayor of Cambridge. The explanation of this rather surprising reception turned out to be as follows: From Aarhus Andrew du Plessis had supplied us with a contact, namely his former supervisor at St. Johns College, George Reid, who in the mean time had become the mayor of Cambridge.

During coffee in the Member's Room he showed us the antique mayor's chair, a somewhat large contraption obviously not meant for people of ordinary size. However Jørn (198 cm.) unlike the mayor made a pretty good fit.

After the reception the mayor led us out onto the balcony overlooking the market place and into his parlour. This caused some amusement for people in the market. In his office we were shown the fate of the gifts which we were later to present to him. He had a huge assortion of presents from visitors from all over the world.

Afterwards we were led into the Council Chambers where we were shown the relics of the town, comprising of the Queen's mace dating from 1710, weighting 12 lb. This mace, which is still used in ceremonies, was rather impressive unlike the reduced mace of the beheaded King Charles (1649). Furthermore the mayor's and sergeant's robes were displayed on live mannequins from the audience (Nina and Jacob G.).

We also saw the original documents giving Cambridge its city rights, which were actually first won in 1951 although the town dated back to 1217 and the first college was founded in 1284.

St. John's College

Next we visited St. John's College (founded in 1511 by Lady Maggy Beauford, mother of Henry VII.) where we were guided around the library and the chapel.

The library dated back to 1623 and the upper floor still contained the original bookcases and books which the college had collected in the years

Figure 27: Jørn and George Reid.

1628–1800. The lower floor is used as a common library.

The first chapel in the college was built in the 13th century, but of this only a few arches and some carpentry remains, having been reused in the new chapel. The new chapel was build in 1863-69 in the Victorian era. The expenses almost ruined the college, as the donator of the chapel tower was run over by a train before he had legally donated the money.

After this we had lunch in the Wordsworth Room, where we were joined by some undergraduates and a couple of college fellows. The students later guided us around in Cambridge.

The sightseeing trip took us through the gardens of St. John's, where

Figure 28: Nina, Jacob G. and George Reid.

modern English architecture had almost succeeded in ruining an astonishingly beautiful old site, and into Trinity college, the former residence of sir Isaac Newton. Also this college had a beautiful courtyard, and an old library where many papers are on exhibition, e.g. the original manuscript of Winnie-the-Pooh, the first edition of Newton's Principia, letters by Goethe, Wittgenstein, Russell etc. Leaving Trinity through the main gates we passed an appletree under the windows of Newton's former room, which is supposed to be descended from a tree in his parents garden, under which he had probably often sat.

Afterwards we were lead to the Kings College Chapel, which lay like an oasis of peace and beauty at the heart of Cambridge; the atmosphere was further enhanced by the music floating from an organ.

Above the altar in the chapel was a painting of magnificent beauty by Ruben, recently donated to the chapel. Finally, after walking through some of the oldest buildings (13'th century) in Cambridge, we crossed a

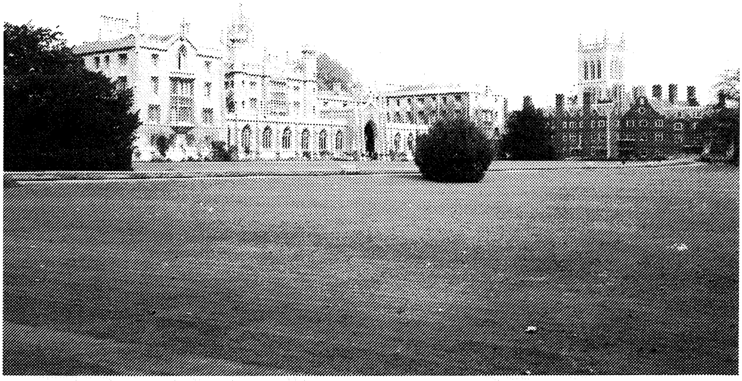


Figure 29: St. John's College.

wooden bridge designed by Newton to stand without the use of bolts.

Unfortunately, curious engineers in their eagerness to understand this exceptional construction had disassembled the bridge, and failing to comprehend the mechanics involved they had had to use bolts in the reconstruction.

Once across the bridge we split into two groups, one of which went to see the Mathematical Institute, the other to see a student's residence at Trinity College.

The student rooms were situated in the older buildings on the ground floor, with windows to the courtyard, and were each shared by two students, each having a smaller bedroom for themselves and sharing a common living room. Despite that the rooms were not modernized it was regarded as a privilege to live in these rooms, and they were occupied by the students with the most seniority.

The Mathematical Institute was in no way a match for the College

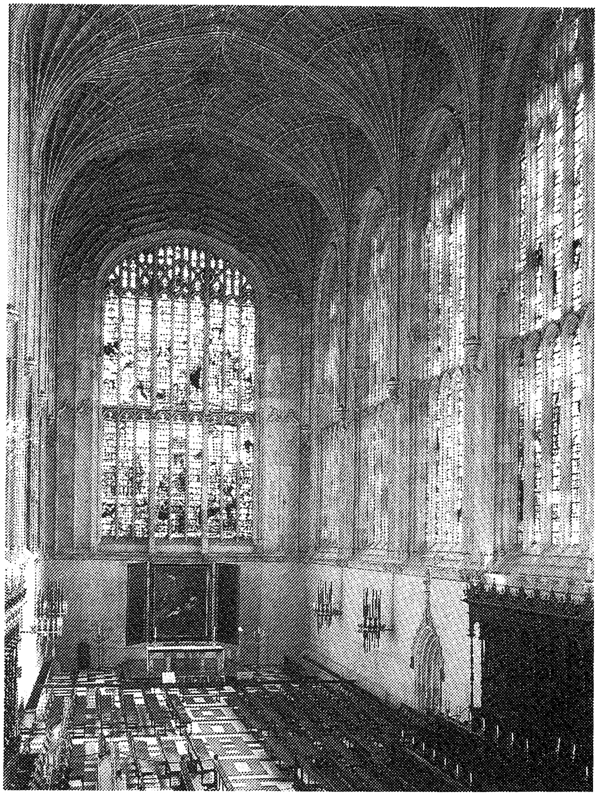


Figure 30: Kings College Chapel.

grounds. For some reason it had been situated in an old factory building, resulting in dim and obscure corridors in which no people were to be seen. Inspite of this the academic staff counted several names of significance. The remoteness of the site was best illustrated by the fact that our guide was unaware of its location.

Figure 31: Newtons Bridge.

Conclusion

We conclude that our visit to Cambridge consisted mainly of sightseeing and did not involve many things of explicit mathematical interest.

Finally we would like to thank the mayor for his cordial welcome and David Quest and friends for their interesting and humorous tour through Cambridge.

Open University

On October 25, 1990 we visited the Open University in Milton Keynes, situated between London and Birmingham. The programme was as follows:

11:00	Arrival, coffee in the refectory.
11:30	Tape slide presentation of the Open University, followed by a guided tour around the University.
12:30	Lunch
2:45	A discussion with members of the O.U. staff
3:30	Tea and departure.

Unfortunately, a visit at the BBC studios had to be postponed and finally cancelled. Instead we proceeded with the discussion with the staff.

The report consists of the following parts:

- The Open University in general: history, statistics and the basic structure of the O.U.
- The guided tour around the university, featuring a lot of pretty photos.
- The discussion with the staff: structure of math programmes, designing new courses and data concerning the mathematical faculty.
- Discussion of different aspects of the O.U.
- Conclusion of the visit.

General Facts about Open University

The Open University is situated in Milton Keynes, in precisely the center of a linesegment connecting the cities London and Birmingham. It was established in 1969 and is now Britain's largest single institution, with plans to expand into Europe.

More than 100.000 students each year enroll in the Open University, and a similar number buy self-contained study packs. Thus in any given year more than 200.000 people are likely to be involved in courses in the Open University. The most well known feature is the undergraduate programme, which has approximately 72.000 current students and more than 102.000 successful graduates.

The modular nature of Open University courses allows students to construct their own degree programmes in accordance with their aptitudes, interests or career requirements. It also permits thousands more to take advantage of Open University courses each year on a 'one-off' basis, as associate students.

The University has a higher degree programme which includes full and part-time degree studies, consisting either of courses or research activities. The University also places considerable importance on the research activities of its academic staff.

In addition to the undergraduate programme, the Open University has greatly expanded its service to the wider community. In the fields of education, health and social welfare, advances in science and technology, family, personal and cultural education and leisure interests, the Open University offer diplomas, short courses, and scores of study and training packs. In a given year it expects to sell around 80.000 of these self-contained packs, many of which are used by more than one person. In 1983 an Open Business School was established within the University, based on the same principles.

No entry qualifications are required for any Open University courses, besides the higher degree courses. The only requirement is that applicants must be adult residents in the UK. Limits on staffing and resources determine how many students can be accepted on each course, so there are sometimes quotas and waiting lists. In most years, applications for entry to the undergraduate programme outnumber the places available by more than two to one.

The characteristic feature of all Open University teaching is that it takes place at a distance. The student learns in his or her own home, or sometimes at their place of work. All courses are designed for students learning on their own.

At the core of almost all courses are a series of specially produced textbooks (known as *units* within the Open University). These are linked in most cases to other educational material: radio and television programmes, audio and video tapes, home experiment kits and computer software.

The TV and radio programs are produced in a special department of the BBC, which is situated on the campus. The programs are broadcasted at off-peak times, but are nevertheless seen by more than 500.000 people. The amount of printed material used in the courses has made the Open University one of Britain's largest publishers of educational material. 35.000 kg is sent out from the University every week, and this is only 10% to the total distribution.

Although students learn at home, there is plenty of support and human contact available. Each undergraduate and associate student has a local tutor and counselor, who they can meet along with their fellow students at one of the network of some 250 Open University study centers throughout the country. (The study centers have a staff of approximately 5000, mostly part time academics, while the university itself has a staff of 2000, 350 of which are academics.)

Students can also meet at residential schools, which are an integral part of many Open University courses. Every undergraduate student must attend at least one summer school, which usually lasts a week; these are held on the campuses of universities around the UK during their long vacation.

When it is possible students meet regularly for studygroup sessions. The amount of work expected of a student varies from course to course. A typical undergraduate full credit course involves 12–14 hours study per week over a nine-month period.

The largest single group of Open University students are those studying for the BA degree (nearly 72.000 in 1990). The Open University now produces nine per cent of all the first degree graduate from UK universities each year.

Students choose from a range of 140 courses at foundation, second, third and fourth level, offered by the University's six faculties or schools: Arts, Social Sciences, Education, Mathematics, Science and Technology. The courses are rated as full or half credits, depending on the amount of work involved.

Six credits are required for a BA degree and eight for a BA Honors degree. Students are not allowed to take more than two full credit courses in a single year, making three years the minimum period for completion of BA studies. Most students find that one-and-a-half credits is the most they can comfortably combine with family and work commitments, and therefore take between four and six years to graduate.

The assessment of students is rigorous, consisting of continuous assessment of assignments during the course and a three hour supervised examination. External examiners ensure that Open University standards are comparable with those of other British universities.

The majority of students are in their late twenties, thirties, or forties. The youngest students are in their late teens, the oldest graduate (so far) is over ninety. Almost half of all Open University students are women - the highest proportion of women students at any UK university. Three out of four Open University undergraduate and associate students are wage owners throughout their studies. Despite the unrestricted admission policy, about 70% of the undergraduates successfully complete their first-year examinations, and more than 55% successfully graduate four, five, six or more years later.

The university offers courses in a wide range of specialized topics for teachers, health service and social welfare workers; these are available either to individuals or as block bookings for organizations. A relatively new development for the University is a programme consisting of course work leading to the Masters degree in mathematics. The University also offers opportunities to take research-based B.Phil., M.Phil. and Ph.D. degrees involving full- or part time study. There were 650 research students enrolled in 1989.

The Open University has a budget of £125.000.000 per year. The Government provides £80.000.000 of this, while undergraduate fees amount to £15.000.000. These fees, which lie in the region of £2000 for a BA-degree, do not represent the full production cost of the courses. Short courses on the other hand are fully paid for by the students or their employers. Courses are expensive to create, but relatively cheap to operate, thus, due to the many students, unit costs at the Open University are well below those of conventional universities. Open University costs per undergraduate student - in full-time student equivalent terms - are about 40% lower.

A Resume of our Guided Tour of the Open Universities

After a coffee-break we where shown around the grounds of the Open University by John Deeley, the Visits Officer. We began with the Visitors Office. In order to prevent overcrowding we were divided into 2 groups, who took it in turns to see a half-hour tape-slide presentation of the Open University. While one group was seeing the presentation the other was shown around the university campus.

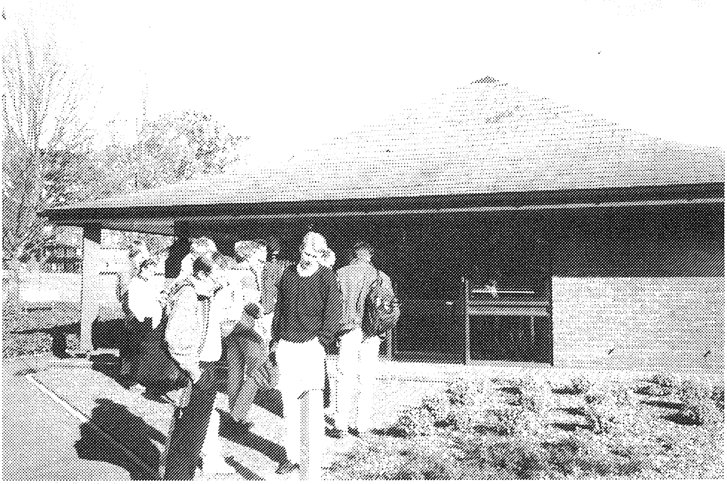


Figure 32: Here we see some of us waiting outside the Visitors Office.

We began by walking across the campus shown at the picture below. First we were shown the building where the Open University was started in 1969. Unfortunately we didn't get a picture of this building.

Afterwards we walked through the building in the background of the picture above. We saw that a grocery shop and bank were situated on the campus. We also noticed that some of the buildings consisted of prefabs (see picture below), a sign that the expansion of the Open University had outstripped the rate at which new buildings could be built.

Finally we where shown the buildings where the BBC is situated, and the place from which the printed material is sent out each week by truck.

Figure 33: The campus seen from the Visitors Office.

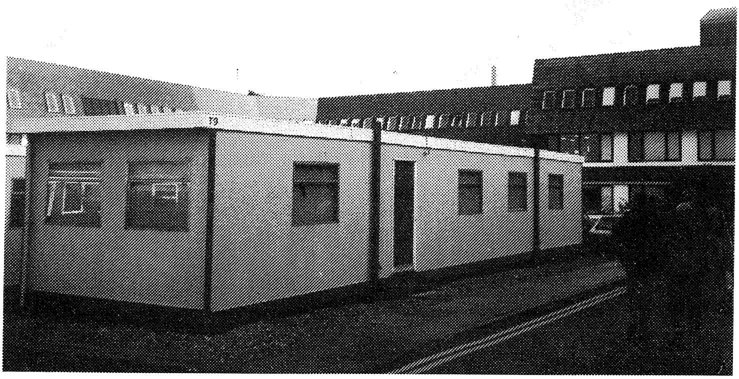


Figure 34: A picture of one of the prefabs.

The meeting with the staff.

The meeting was held in the beautiful 600 year-old church, shown in figure 35.

Figure 35: The old church where the meeting was held.

Mr. Jeremy Gray, who was our contact person, and the two members of staff Robert Wilson and Michael Adams started by telling us some facts about the structure of the mathematics degrees at the Open University, which have been obtainable since 1971. Afterwards the meeting took the form of a dialogue between the staff and us. There are three levels of undergraduate courses at the university. They are most easily described by giving examples of what they consist of.

Level 1 courses:	(Foundation course) Basic calculus etc.
Level 2 courses:	Pure Maths: Linear algebra, group theory, geometry, real analysis, advanced calculus. Applied Maths: Modeling, real life applications, differential equations, computing, statistics, probability theory. History of mathematics.
Level 3 courses:	Number theory, various forms of topology, graph theory, abstract algebra (Galois), Lebesgue integration, differential geometry.

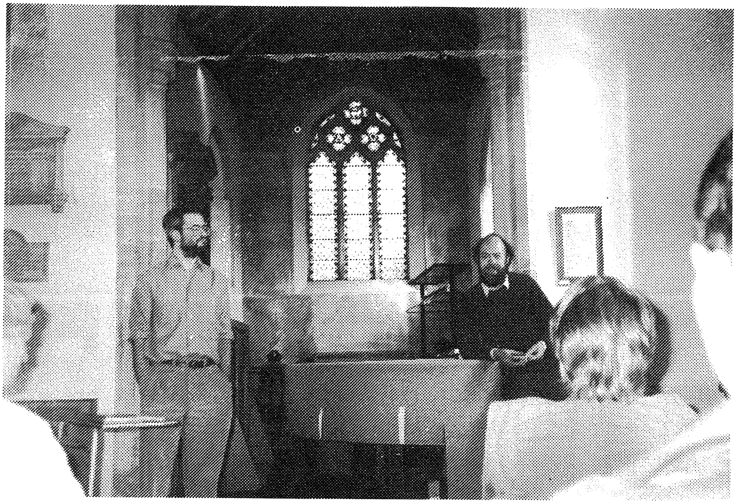


Figure 36: Mr. Gray and Mr. Michael Adams who gave the presentation.

After completing their BA degree students can go on to take a M.Sc., M.Phil. or Ph.D. degree. For this they need closer, more personal contact with a supervisor, either at Milton Keynes or in the vicinity of where they live.

After the presentation of the maths programmes, Mr. Wilson told us about how new courses are made. As an example he told us about a graph theory course – an interfaculty course made in cooperation by the Maths, Science and Technology faculties. The initial idea for the course came from discussions between members of these faculties. These resulted in the formation of a team consisting of faculty members, people from the BBC and other relevant people, altogether 12 people, who developed the course over a three year period. Different members of the team made suggestions for the course material and these were discussed and evaluated by the team as a whole. They finally ended up with a complete set of course material, which among other things consisted of 16 television programs. The total cost of the project was about £500,000.

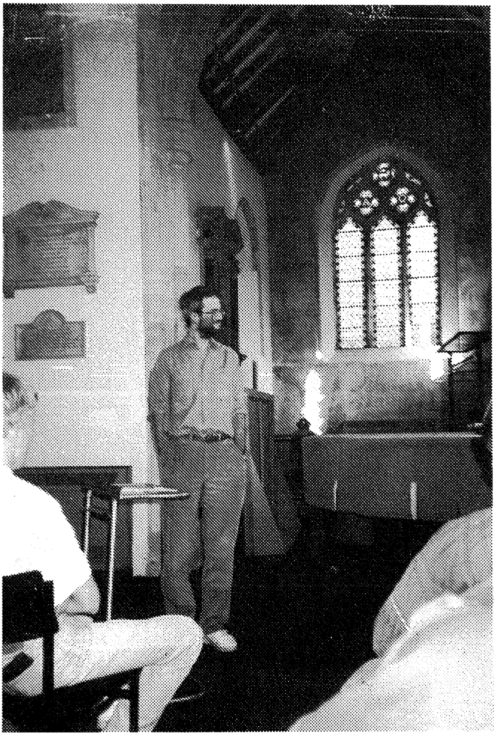


Figure 37: Mr. Wilson, son of the former prime minister, Harold Wilson.

The TV programmes are not just televised normal lectures, but try to use the media to illustrate things that would normally be hard to do on a blackboard. As an example there are no TV programmes in group theory, whereas a subject like differential geometry has an obvious potential for TV display. Normally about 60 people are involved in making a programme, which on average costs about £10,000.

After this we were presented with some facts about the maths faculty. The faculty has an academic staff of 80 people, whereof 30 are working at Milton Keynes, who give lectures, write textbooks and notes, examine the students and do research projects. They get about two months annually for research. In addition to this there are about 100 regional tutors spread across the U.K., who mark assignments from the students and actually meet them face to face at the local study centers. These sessions take place about once a week at the foundation course level and every two or three weeks later on. Every summer there are summer schools, where the students stay for a number of weeks. This is the only chance the staff gets to actually meet the students and get feedback on the courses. There are plans, though, to introduce some kind of student evaluation of the courses.

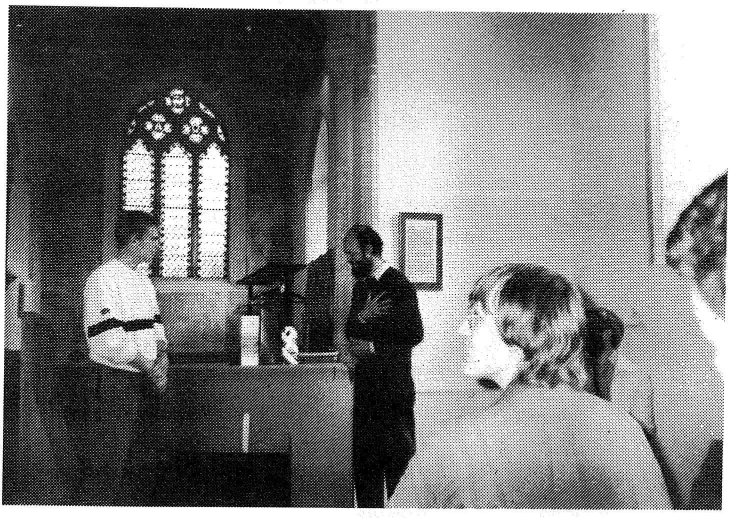


Figure 38: Mr. Gray receives the presents from "Friends of Euler".

Finally, in the refectory, we were given the chance to look at some of the books which are used in the maths courses. In our view, they were readable and illustrative.

Discussion

There are of course advantages and disadvantages in the form of studying required for O.U. courses. The advantages are that you can study at home, whenever you want and at your own speed (half/full credit). Thus you can have a job or other obligations (family etc.) while studying. Furthermore you don't have to have A-levels to be admitted to the O.U. As a result of this a lot of students who would otherwise not have obtained a university degree, now do so. The maturity and independence of students is increased, simply because they have to work more on their own. This is a valuable quality for many firms. With respect to the education methods, it is sometimes possible to be more illustrative on a TV screen than on a blackboard. It is also possible to hear an argument repeated, simply by rewinding the tape.

The disadvantages are that students don't have daily personal contact with the tutors. This means that they must wait up to three weeks to ask questions, which might demand an immediate answer (they can of course call the tutor, but few do). They can't give feedback to the courses, except by writing. The social life among students is more restricted, especially when they are living far away from a study center, so they can't discuss problems as easily with other students. This could lead to many students feeling isolated. Because of the less demanding prerequisites some students have to do more work to reach the same standard as the other students. For subjects requiring experimental equipment, like physics and chemistry, there is also the problem of giving the students access to these things. Financial limits restrict the amount of equipment that can be mailed to the students.

As a conclusion we think that the O.U. is an excellent alternative to the existing universities, and that both systems have a justification within their own "target" areas. We think that it would be a good idea to have a similar institution in Denmark, perhaps in cooperation with the O.U.

Evaluation and conclusion

We were very pleased to visit the O.U. It was interesting to learn about a totally different way of teaching students than the one we've been used to.

The discussion with the teachers was very interesting and informative. We were disappointed to learn that we weren't able to visit the BBC studios, as this was an important part of the programme. But all in all the visit was a good supplement to the picture we had gained of the British educational system.

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